second space constructed by James, known as his tree space, gave a counterexample to another question of Banach by having a non-separable dual but containing no subspace isomorphic to ℓ_1. (This problem was independently solved by Lindenstrauss and Stegall, using a function-space variant of James's space.)

In their comprehensive book Fetter and Gamboa de Buen give complete solutions to these and other problems and prove a variety of properties of the James spaces. They are keen for the book to be accessible to graduate students and for this reason most of their arguments are given in complete detail.

Occasionally their efforts at accessibility can be counterproductive. In particular, they have a tendency not to distinguish between trivial facts and more substantial ones; sometimes it is easier to be told to prove something yourself than it is to follow somebody else's argument. There are many examples of this, of which here is a small sample. On page 14 they attribute to Singer the result that there exists a space which embeds into its double dual with codimension k. (Such a space is called quasi-reflexive of order k.) Here I shall leave the result as an exercise (given what has been said already). On the next page they attribute to Bessaga and Pełczynski the solution of the first Banach problem mentioned above, quoting from a paper of theirs (published ten years after that of James) the result that, if X and Y are quasi-reflexive of order n and m, then $X \oplus Y$ is quasi-reflexive of order $n+m$. This, it seems, was the missing step needed to show that James's space was indeed not isomorphic to its square. On page 28 they need the fact that finite representability is transitive. This is more or less immediate from the definition, but they refer the reader to a book of Beauzamy. This kind of thing can only confuse a graduate student. (Incidentally, on the previous page they appear to confuse Dvoretzky's theorem with the Dvoretzky-Rogers theorem.)

Despite these faults the book will be a very useful reference to anybody who feels that a James-type space may help them to solve one of their problems. In this it will play a similar role to the book of Casazza and Shura on Tsirelson's space (Lecture Notes in Mathematics Vol. 1363, Springer-Verlag, 1989).

W. T. GOWERS

DAVID, G. and SEMMES, S. *Fractured fractals and broken dreams – self-similar geometry through metric and measure* (Oxford Lecture Series in Mathematics and its Applications Vol. 7, Clarendon Press, Oxford, 1997), ix + 212pp., 0 19 850166 8, (hardback) £35.

A great many pages have been written about fractals that satisfy the very strong condition of strict self-similarity, where small regions are directly similar to the whole set. Nevertheless, it is easy to construct fractals where parts of different regions resemble each other in a rather weaker sense. For example, if a self-similar set undergoes considerable distortion or breaking, much of the fine-scale geometry will still remain. This book presents a new and wide ranging theory for analysing sets with different parts at different scales looking "vaguely alike".

A set or space is said to satisfy the BPI "big pieces of itself" condition if, given any pair of balls centred in the set, there are substantial subsets within the balls that look roughly alike in the sense of Lipschitz equivalence. There is a similar definition for the BPI equivalence of two sets. The weaker, but related notion of one set "looking down" on another is also central. The book is devoted to developing and applying these notions. For instance, one basic result is that two BPI sets are BPI equivalent if they contain a pair of subsets that are bi-Lipschitz equivalent.

The book is full of examples and constructions of BPI sets, including familiar fractals, Heisenberg groups, nested cubes constructions and deformations of sets. Thus the theory embraces a far wider range of fractal sets than the now familiar iterated function system framework. The development includes results on weak tangents, measure properties and Lipschitz mappings.

Most of the material in this book is completely new and the style, though unusual, is a refreshing change from conventional texts. The authors have taken a natural but not too strong notion relating to sets of fine structure, and followed through its properties, relationships and applications. They freely admit that their framework is not the only possible one, but by the end of the book they have more than justified their claim that their approach is both rich and flexible. Certainly, they have only explored a few of the possible questions; further directions are indicated by the many explicit open questions and others readily suggest themselves to the reader.

The book is accessible to anyone with a modest knowledge of metric spaces and measure theory, with other background material being introduced as and when required. The theme of "big pieces of itself" leads the reader on a tour through a range of attractive areas of mathematical analysis, from geometric measure theory to manifolds, and from quasiconformal mappings to dynamical systems. The book is recommended not only for those interested in the broad subject of the geometry of fractal sets and measures but also as a fine insight into how two eminent mathematicians explore and develop a new area.

K. J. FALCONER