66.F Here A is a point outside the circle, AM and AN are tangents, and the lines APQ, ARS meet the circle at P, Q, R and S. Prove that the point of intersection of PS and RQ lies on the line MN.

As D. M. Hallowes pointed out to me, this was like asking him to prove that $2 + 2 = 4$. And many of you (G. A. Garreau, J. Clemow, R. F. Cyster, C. E. A. Burnham, James Petty, T. Knape Smith, Bruce Andrews and others) sent in one-line projective proofs (or their equivalents). I had expected that, by restricting attention to the circle, there would be a lot of proofs avoiding projective geometry. The fact that I was mistaken is perhaps further testimony to the power of that subject. But amongst non-projective proofs (from Daphne Medley, E. L. Russell, Chris Bishop and others) I briefly quote the solution due to H. Matley and S. E. Eldridge.

"If the circle is $x^2 + y^2 = r^2$, A is $(a, 0)$, P is $(r\cos A, r\sin A)$ and Q is $(r\cos B, r\sin B)$, then it can be shown by coordinate geometry that the point of intersection required is:

$$\left(\frac{r^2}{a}, \frac{r(a^2 - r^2)(a\sin(A + B) - r(\sin A + \sin B))}{a((a^2 + r^2)(\cos A + \cos B) - 2ar(1 + \cos A\cos B))}\right) \text{and } MN \text{ is } x = \frac{r^2}{a}."$$

Thanks, once again, to you all. Keep writing!

Correspondence

Sums of two squares in three ways

DEAR EDITOR,

In note **66.9** H. M. Finucan lists some numbers which can be expressed as the sum of two squares in at least three different ways. All his examples are multiples of 5 but in answer to his question this need *not* always be the case. What is more significant about his examples is that they all have three prime factors of the form $4n + 1$ with at least two of the three different. The smallest number of this kind that does not have 5 as a factor is $13^2.17 = 2873$. And in that case $2873 = 8^2 + 53^2 = 13^2 + 52^2 = 32^2 + 43^2$.

Yours sincerely,

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Sums of two cubes

DEAR EDITOR,

In an article in the March 1982 edition of the Gazette I considered those numbers n for which there exist integers a, b and c with

$$n.a^3 = b^3 + c^3.$$

The only numbers less than 100 not dealt with were 60, 66, 73 and 94.

Professor Cassels has kindly written to me pointing out that there are no solutions for 60, 66 and 73, but that

$$94 \times 590736058375050^3$$
$$= 15642626656646177^3 - 15616184186396177^3.$$

Now why didn't I spot that?

Yours sincerely,

STAN DOLAN

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