Abstracts of Australasian Ph D theses

Asymptotic estimates of the errors in the numerical integration of analytic functions

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An expression in terms of a contour integral for the remainder term $R_{n+i}(f)$ in the general numerical integration formula,

$$\int_a^b w(x)f(x)dx = \sum_{k=0}^n \lambda_{k,n}\, f\!\left(x_{k,n}\right) + R_{n+1}(f)\ ,$$

is obtained. The function $f(x)$ is allowed to have singularities at ·the end points of the interval of integration and therefore, special emphasis is placed on the behaviour of the contour in the neighbourhood of these points. From the general expression the corresponding contour integral forms of the remainder in some of the well-known integration formulae are derived.

The contour integral, together with known asymptotic expressions for large n of part of the integrand, is used firstly to examine the convergence properties of certain integration schemes and secondly, to obtain realistic estimates of the errors in approximating to a real integral by a weighted sum of values of $f(x)$.

The expressions are also used to obtain useful upper bounds for the errors.

In conclusion some related problems are discussed; for example, a way in which new numerical integration formulae may be obtained is suggested and a unified treatment to numerical integration formulae is given.