ON THE FORMULA USED BY MR. JELLICOE IN THE GRADUATION OF THE EAGLE INSURANCE COMPANY'S EXPERIENCE.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Having recently had occasion to examine the method of graduation described by Mr. Jellicoe in his valuable paper "On the Rate of Mortality in the Eagle Insurance Company", in vol. iv of the *Journal*, I found that no demonstration was given of the working formula, as set forth on p. 206. In a footnote on that page, Mr. Jellicoe says, "The mode of obtaining this expression I defer till a future occasion, when I propose to examine all the methods that have been devised for the purpose in question." As the proposed explanation has not been published, a short demonstration of the formula will probably be considered of sufficient interest, even at this interval of time, to justify its insertion in the pages of the *Journal*. At any rate, future students will be saved some time and trouble in obtaining the formula for themselves.

The method of graduation is as follows:—If $\Sigma q'_x$ denote the sum of the unadjusted rates of mortality for n successive ages beginning with age x, and $\Sigma q'_{x+n}$ denote a similar quantity beginning with age $x+n$, then the adjusted rate of mortality for ages x to $x+2n-1$ inclusive, is made to increase in geometric progression, the common ratio r being found from the equation,

$$r^n = \frac{\Sigma q'_{x+n}}{\Sigma q'_x} = p$$

$$\text{or} \quad r = p^{\frac{1}{n}}.$$

Taking, now, a as the starting age, and denoting the adjusted rate of mortality at that age by q_a, we have

$$\Sigma q'_a = m = q_a + q_{a+1} + q_{a+2} + \ldots + q_{a+n-1}$$

$$= q_a + p^{\frac{1}{n}} q_a + p^{\frac{2}{n}} q_a + \ldots + p^{\frac{n-1}{n}} q_a$$

whence

$$q_a = \frac{m}{1-p}\left(1 - p^{\frac{1}{n}}\right)$$

$$q_{a+1} = \frac{m}{1-p}\left(p^{\frac{1}{n}} - p^{\frac{2}{n}}\right)$$

$$\ldots\ldots\ldots$$

$$q_x = \frac{m}{1-p}\left(p^{\frac{x-a}{n}} - p^{\frac{x-a+1}{n}}\right)$$

$$= \frac{m}{p-1}\left(p^{\frac{x-a+1}{n}} - p^{\frac{x-a}{n}}\right)$$

$$= 10^{\log\frac{m}{p-1} + (x-a+1)\frac{\log p}{n}} - 10^{\log\frac{m}{p-1} + (x-a)\frac{\log p}{n}}$$

$$= \Delta 10^{\log\frac{m}{p-1} + (x-a)\frac{\log p}{n}}$$

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which is the formula as given on p. 206, with the slight difference of $p-1$ in lieu of $1-p$. This error in Mr. Jellicoe's formula did not, however, affect the results, in consequence of $\log \frac{m}{1-p}$ having been taken as equivalent to $\log \frac{m}{p-1}$.

I am, Sir,
Your obedient servant,
WM. SUTTON.

21 *Fleet Street,*
16 *September* 1874.

MR. HARDY'S VALUATION TABLES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—It may be useful to point out that at page 9 of Mr. Hardy's *Valuation Tables*, $\log a_{55}$ is printed as ·028563 instead of ·082563, and $\log a_{83}$ as ·849818 instead of ·489818.

I am, Sir,
Your most obedient Servant,
GEO. M. LOW

English and Scottish Law Life Assurance Office,
120 *Princes Street, Edinburgh,*
14 *September* 1874.

Mr. E. Justican has communicated the following correction in the same work:—

Page 22, $x+n=50$, $P_{29.\overline{21}|}$ should be ·03900 and not ·03991.