<u>University Calculus with Analytic Geometry</u>, by C. B. Morrey, Jr. Addison Wesley Publishing Company, Reading, Mass, 1962. xiv + 754 pages. $12.50.

Over the past five or ten years many very similar American calculus texts have appeared: 600 or more pages long; covering two years' work; rigorous (in the sense that they mention ϵ and δ) but definitely texts of calculus, not analysis. They have mostly the same vices and the same rather dull virtues, and the prospect of reading through yet another could well be somewhat unappetizing. It is therefore pleasant that the book under review is definitely above the general level. The author has tried to make the book "logically more satisfactory than" most texts, but not to "change the total subject matter in any radical way". I therefore refrain from repeating the list of contents, but comment on the innovations.

The author is very careful about conditions under which formulae are valid. An early theorem states that if $x_1 \neq x_2$ then the line through (x_1, y_1) and (x_2, y_2) has equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

In how many texts is the proviso "$x_1 \neq x_2$" forgotten! In the same vein, we read "If two lines are parallel, they are <u>both vertical or</u> they have the same slope". [My italics.] The treatment is throughout as careful as this, and in my opinion the only possible improvement here would be to use direction-ratios instead of slopes, so making most of the provisos unnecessary.

Between the preliminary work and the calculus proper is a "Review": an intuitive introduction to limits, derivatives, and integrals. This is the kind of thing which any good lecturer gives to his class, but is seldom seen in a text.

Although the author draws attention to his treatment of implicit differentiation in the preface, I did not find this particularly good. The statement

> "The relation defined by the equation $x^2+y^2-4=0$ consists of two functions y defined by the equations $y_1(x) = +\sqrt{4-x^2}$ and $y_2(x) = -\sqrt{4-x^2}$"

is neither elegant, precise, nor illuminating; and there is no mention of local solution, which is the important concept here.

The treatment of greatest and least values is exemplary: not only in text, but even in the examples (and bitter experience shows that this is most unusual) the author proves that his greatest values really are greatest values, and not merely local maxima.

In a valiant attempt to make differentials respectable, the author defines differentials of functions only: starting from $y = f(x)$ he interprets x and y as functions, rewrites the relation as $y(t) = f[x(t)]$, and defines dy as a binary function. However, he then introduces ambiguity by using dy also as an abbreviation for $dy(a, h)$. Of course, taking y as an explicit function of x, instead of having x and y implicitly related, prevents the important use of differentials, namely putting either $dx = 0$ or $dy = 0$ at will to find stationary values of either variable with respect to the other.

An interesting and valuable point is a decent treatment of area at the beginning of the chapter on integration. The treatment of pressure, too, with physical principles clearly stated, is excellent, and points the moral for any application of integration. (The principles are enough to imply that force is the integral of pressure with respect to area.)

Points which particularly pleased me are:-

(i) The proof of the uniqueness of the limit of a given function at a given point before the use of the phrase "The limit" or the notation $\lim_{x \to a} f(x) = \ell$.

(ii) The avoidance of the notation $\lim_{x \to a} f(x) = \infty$. So often a writer takes pains to point out that there is no such thing as ∞, and a little later sets something equal to it!

(iii) The treatment of the approximation to increments by differentials as part of the mean-value theorem, with an estimate of the error in the approximation.

The treatment of parametric and polar equations will please everybody. In particular, the formula in polars for the angle between the radius-vector and the tangent-line contains an additional multiple of π which is usually skipped over, but is here treated properly.

More could be said, but let us just notice that the treatment of partial derivatives is clearer than usual. This reviewer, however, having just written his own book on the subject, will not be expected to admit that it is perfect.

H. Thurston, University of British Columbia

https://doi.org/10.1017/S0008439500026333 Published online by Cambridge University Press