THE EVOLUTION OF ROTATING 15 $M_\odot$ STARS

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Abstract. Theoretical evolutionary sequences have been generated with the YREC code for a 15 $M_\odot$ star from the ZAMS to core helium exhaustion with a variety of physical assumptions, covering both rotating and non-rotating cases. The non-rotating models agree qualitatively with other models found in the literature. The addition of only rotational distortion has little effect on the models, while the full treatment of rotation results in additional mixing and theoretical tracks that are similar to models with small amounts of convective overshoot. Models which include only rotation have fair agreement with the observed main sequence surface rotation velocities, but rotate too rapidly during the post-main sequence phases. The addition of mass loss at the given rates helps this problem somewhat but does not appear to completely resolve it. Neither the non-rotating models nor the rotating models provide full agreement with the terminal-age main sequence band used by Maeder & Meynet (1987); this may be indicative that additional mixing processes are necessary or that a more recent TAMS, such as that of Stothers (1991), should be used.

1. Introduction

An understanding of the evolution of massive stars is important for many areas of astronomy. Massive stars are the most luminous stars; thus, they are the obvious first probes for observational tests of stellar evolution in other galaxies. The progenitors of type II supernovae are highly evolved massive stars, and accurate models of high mass stars are therefore needed to fully understand supernovae. While high mass stars are greatly outnumbered by low mass stars, they give off much more radiation, particularly in the blue and ultraviolet regions of the spectrum. Thus, massive stars strongly affect the UBV colors of galaxies. They are also the sites for heavy element nucleosynthesis, and so influence the chemical evolution of galaxies.

Unfortunately, the evolution of massive stars is at best a poorly understood subject. There are many discrepancies between theory and observation during the main sequence phase of evolution. It is difficult to pick out a "terminal age main sequence" (TAMS) on observed HR diagrams (Blaha & Humphreys 1989, Fitzpatrick & Garmany 1990, and Garmany & Stencel 1992). Theoretically and observationally determined mass-to-luminosity ratios also disagree; observationally determined masses for O and early B stars are in general about 50 per cent smaller than those calculated from models (Herrero *et al.* 1992).

The situation is no better for post-main sequence evolution. The observed blue-to-red supergiant ratio is significantly larger than has been predicted (Chiosi *et al.* 1978; Bressan *et al.* 1981; Brunish & Truran 1982a,b). This is probably related to another problem, that of post-red-supergiant (post-

L. A. Balona et al. (eds.), Pulsation, Rotation and Mass Loss in Early-Type Stars, 131–144.

RSG) blue loops. The "extra" blue supergiants (BSG) may be due to a BSG phase occurring after the RSG phase. This idea is supported by the observation of nitrogen-enriched BSGs (Walborn 1988), as convective dredge-up in the RSG stage can mix nitrogen and helium into the envelope. Other evidence of a BSG stage after an RSG phase comes from observations of the progenitor for SN 1987A, which turned out to be a BSG. Unfortunately, only a few models produce the necessary blue loops, and it is difficult to explain the observations using alternative theories.

Probably the greatest reason for these and many other unsolved questions is that theories of massive star evolution must incorporate a number of physical processes which are highly uncertain. Significant amounts of mass loss are observed in high mass stars, at rates as much as 10^9 times those for solar-type stars (Conti 1978, Chiosi & Maeder 1986). A number of theories have been proposed for modelling mass loss; unfortunately, all of these theories have problems. The best results are obtained from empirical fits to observations, with the mass loss rate expressed as a (hopefully) simple function of stellar parameters (mass, radius, luminosity, etc.). These parameterizations vary in complexity and reliability, and most are limited in use to a specific region in the HR diagram (cf. Chiosi & Maeder 1986 and references therein). Massive stars may also undergo convective overshoot at the edge of the convective core. Since convection is extremely difficult to model in a realistic manner, it is equally problematic to produce convincing models of overshoot, and the results of such attempts vary greatly, from virtually no overshoot (Saslaw & Schwarzschild 1965) to extremely large amounts (Roxburgh 1978, Cloutman 1978). Another process which may be very important in massive stars is semiconvection. Once again, the physical basis of this process is poorly understood, and different models can give extremely diverse results. There has been considerable debate over whether or not semiconvection occurs at all, but the biggest controversy is over which criterion to use for convective neutrality. If semiconvection is a "fast" process, the usual Schwarzschild criterion should be used; if it is a "slow" process, the Ledoux criterion, which includes a term to account for μ gradients, should be used. To further complicate matters, these processes are not independent. Some models of mass loss reduce the amount of semiconvection (e.g., Sreenivasan & Wilson 1978, Chiosi 1981a). Convective overshoot should also affect semiconvection, because it will alter the abundance profiles within the model. Mass loss and overshoot interact in such a manner as to produce very different TAMS from models including only one or the other.

The addition of rotation to this already messy subject may seem like an exercise in futility, but is necessary for a realistic treatment of the evolution of high mass stars. Most stars rotate, and massive stars are no exception. Observations show that massive stars have a mean $v \sin i = 50$–200 km s^{-1}, with a good deal of variation (Fukuda 1982). Our understanding of rotational effects is increasing rapidly. Rotation produces a distortion from spherical symmetry; rotating stars of a given central density tend to have lower central temperatures than their non-rotating counterparts. Rotation can also cause chemical mixing, as unstable angular velocity gradients can occur across regions of varying chemical composition. Both of these effects tend to increase the overall lifetime of the star. In addition, rotation is believed to enhance mass loss, possibly by as much as a factor of 2 to 3. Finally, rotation has been shown to have significant effects on the later stages of evolution for low- and intermediate-mass stars (Pinsonneault *et al.* 1989; Endal & Sofia 1976, 1978). There is no reason to suppose it will not be important for massive stars, as well. Even so, there has been very little work done which incorporates rotation into the evolution of massive stars.

We have chosen a star of 15 $M_{\odot}$ as the subject of this study for a number of reasons. The 15 $M_{\odot}$ is well above the transition mass of stars which can ignite carbon, neon, oxygen, and silicon burning in the core. This gives the possibility of eventually enlarging this work to include evolution beyond the core-helium-burning stage. Some work has been done on rotating intermediate mass stars (less than 10 $M_{\odot}$; see, e.g., Endal & Sofia 1976, 1978), but higher mass stars thus far have been neglected.

2. The Evolution of Massive Stars: An Exercise in Uncertainty

2.1. SEMICONVECTION

The existence of semiconvective mixing in massive stars was first proposed by Schwarzschild & Härm (1958). Semiconvective mixing results from an instability brought on by the dominance of electron-scattering opacity in the interiors of stars with masses greater than about 10 $M_{\odot}$.

Since the introduction of the concept of semiconvection, a great deal of work has been done to try to answer two questions: is this an important process in the evolution of massive stars, and does it matter which criterion is used to determine convective neutrality? This problem is in a sense a question of whether semiconvection is a "fast" or a "slow" process. If it occurs relatively rapidly, the composition gradient is quickly smoothed out and the Schwarzschild criterion is appropriate. If, on the other hand, the process is relatively slow, the stabilizing mean molecular weight gradient will remain for some time and the Ledoux criterion should be used.

Recent observations, as well as calculations using YREC and other codes, may provide more clues to the answer to this dilemma. Blue loops are one possible solution to the discrepancy between the predicted and observed blue-to-red supergiant ratios. Observations of nitrogen-enriched BSGs by Walborn (1988) support the existence of these loops, as convective dredge-up during the RSG phase can mix nitrogen and helium into the envelope. In this light, use of the Ledoux criterion and "slow" semiconvection is probably more appropriate, as these models are more consistent in producing blue loops. SN 1987A also provides evidence in favor of this.

2.2. CONVECTIVE OVERSHOOT

The classically defined edge of a convective region is the point at which $\nabla_{rad} = \nabla_{ad}$. Within the boundaries of this region, complete mixing occurs due to the turbulent motion of the material. At the edge of the region, the material is said to be stable against convection. However, it is not clear to what extent this boundary provides an end to mixing, and this point has important consequences on the evolution of the star.

2.3. MASS LOSS

Mass loss is a common feature of all stars of high luminosity. Theories for the physical mechanisms behind mass loss can be divided into two regions in the H–R diagram: those for early-type stars, and those for late-type.

There has been a wealth of evolutionary calculations performed for massive stars with mass loss (see reviews by de Loore 1980, 1981, 1982; Chiosi 1981a,b; Chiosi 1982; Maeder 1984a,b; and Chiosi & Maeder 1986, and references therein.) The mass loss rates are generally expressed as empirical formulations, and the mass of the model is decreased with time according to the assumed rate. While this approach is not entirely realistic and may neglect important aspects of the mass loss process, it avoids the problem of implementing a poorly understood physical model.

2.4. ROTATION

Rotation directly affects the structure and evolution of a star in two ways. It causes departures from spherical symmetry through the addition of centrifugal forces; this is sometimes referred to as the structural effects of rotation. Rotation can also give rise to both secular and dynamical instabilities, which act to redistribute angular momentum throughout the star. The redistribution of angular momentum will affect the structure via rotational distortion, and the associated mass motions will cause chemical mixing which can have profound effects on the evolution. Reviews on this subject include Tassoul (1978, 1984, 1990), Kippenhahn & Thomas (1981), Zahn (1983), and Schatzman (1984).

Several techniques for computing rotating stellar models have been developed over the years, with varying degrees of success. Kippenhahn & Thomas (1970) derived a method which was then adapted by Endal & Sofia (1976). This technique uses the mass contained within an equipotential surface, M_ψ, as the independent variable. The angular momentum distribution is used to solve for the shape of the equipotential surface, and distortion terms are calculated which are added to the standard stellar structure equations. The rotation-modified equations are then solved in the normal way. A modified version of this was devised (Endal & Sofia 1978, 1979) which attempted to solve the problem of angular momentum redistribution by various instabilities (cf. Endal & Sofia 1978 and references therein). Using Prather's (1976) code (modified by Seidel *et al.* 1987) as a starting point, Pinsonneault (1988) created the Yale Rotating Evolution Code (YREC), which incorporates Law's (1980, 1981) treatment of structural distortions and Endal & Sofia's (1976, 1978) concepts of rotationally-induced mixing. This technique has been applied to rotating models of the Sun (Pinsonneault *et al.* 1989) and, using solar calibrations, to the study of young open clusters in the Galactic disk (Pinsonneault *et al.* 1990) and low metallicity halo stars (Deliyannis *et al.* 1989; Deliyannis and Pinsonneault 1990; Pinsonneault *et al.* 1991, 1992). The studies using this code have been very successful in matching both global rotational and composition constraints. While this does not guarantee that all of the assumptions used in this method are correct, the technique has considerable predictive power and flexibility. It has been used with great success for stars with a variety of masses, metallicities, and evolutionary status. Of course, there are always unanswered questions, and so YREC is a continually evolving code.

3. The Evolution Code

The most recent version of YREC includes new opacities, up-to-date nuclear reaction rates, and improved formulations for rotationally-induced instabilities (Pinsonneault 1993). The code used for this research also includes mass loss and atmosphere tables generated from the latest Kurucz atmospheres (Kurucz 1992).

3.1. EQUATION OF STATE

YREC uses a simplified equation of state with three regimes. In regions where $\log T < 5.5$, the Saha equation is used to solve for a single ionization state of hydrogen and heavier elements, and the single and double ionization states of helium. When $\log T > 6.0$, full ionization is assumed, and an iterative equation of state calculation is performed using tabulated values for the electrons, which are partially degenerate and partially relativistic. A smoothed interpolation scheme is used in intermediate regions. For more details, see Prather (1976).

3.2. OPACITIES

A number of different opacity tables are available for use with YREC. This work used Kurucz molecular opacities (Kurucz 1991) for low temperatures and the Lawrence Livermore (OPAL) opacities (Iglesias & Rogers 1991, Rogers & Iglesias 1992) at all other temperatures. These two sets of opacities are considered the best available today. The two sets are averaged in the temperature range $4.0275 \leq \log T \leq 4.0325$, using a weighted ramp function to ensure a smooth transition between the tables.

3.3. NUCLEAR REACTIONS

The energy generation calculations include the individual rates for the PP chain, the CNO cycle, the triple-alpha process, and neutrino losses. The reaction rates in YREC have been checked and revised by John Bahcall so that they agree with the modern numbers in Bahcall (1989) and Bahcall & Pinsonneault (1992). The energy released per reaction has been updated to agree with the values of Bahcall & Ulrich (1988). In addition to these changes, Marc Pinsonneault has also introduced an implicit nuclear burning scheme which uses weighted mean reaction rates in convective cores.

3.4. CONVECTION AND THE MIXING LENGTH THEORY

The Schwarzschild criterion is used to determine stability against convection. Convective regions are assumed to be instantaneously, homogeneously mixed. If $\log T < 6.9$ in a convective region, mixing length theory is used to calculate the temperature gradient (see Prather 1976 for details); otherwise, the temperature gradient is assumed to be adiabatic.

3.5. SURFACE BOUNDARY CONDITIONS

One method of generating surface boundary conditions is to compute an Eddington grey model atmosphere. This works in most regions of the HR diagram, but can prove costly in computational time; also, there are some regions of the HR diagram where this technique as applied in YREC is problematic for massive stars. Tables of surface boundary conditions were constructed to resolve this difficulty.

3.6. SEMICONVECTION

YREC contains a formulation for helium semiconvection developed by Castellani *et al.* (1971) for the cores of horizontal branch stars. It is not clear if this is applicable to the helium-burning convective cores of massive stars. Preliminary studies showed that the extent of additional mixing is extremely small for blue and red supergiants, and even small amounts of extended mixing due to convective overshoot or rotationally-induced instablities eliminate it entirely. Thus, this effect is ignored in these models.

YREC contains no formulation for hydrogen semiconvection. Since extended mixing due to convective overshoot or rotationally-induced instabilities probably eliminates most, if not all, semiconvection, it was not added to the modified version of YREC used for this project.

3.7. CONVECTIVE OVERSHOOT

Theoretical models of convective overshoot contain a great deal of uncertainty. The best results have been obtained by assuming arbitrary amounts of overshoot and attempting to match models to observations. The amount of convective overshoot is thus specified in the form of some fraction of the pressure scale height at the edge of the convective region, and material is mixed out to this distance. Three different overshoot parameters can be specified for extended mixing above a convective core, below and above an intermediate convective zone, and below a convective envelope. The temperature gradient is assumed to be non-adiabatic in the overshoot region; thus, this is truly convective overshoot as per the nomenclature of Zahn (1991).

3.8. MASS LOSS

Mass loss has been incorporated into YREC in a fairly simple manner: mass is removed from the stellar model during each time step at a rate supplied by the user. The mass loss rate is given in the parameterized form

$$\log \dot{M} = C_1 \log M + C_2 \log L + C_3 \log R + C_4 \log g + C_5 \log \eta + C_6 \qquad (1)$$

where C_1, C_2, C_3, C_4, C_5, and C_6 are constants, η is as per Reimers' formulation (Reimers 1975), and all other quantities are as usual, in solar units.

3.9. ROTATION

The treatment of rotation incorporated into YREC is conceptually the same as that used by Endal & Sofia (1976, 1978) but the technical details are different, as YREC was re-written using Prather's (1976) code as a starting point. YREC differs from all non-rotating codes excepting that of Endal & Sofia in that it accounts for the effects of both non-spherical geometry and the transport of angular momentum due to rotationally-induced instabilities. Details of this method are given in Endal & Sofia (1976) and Law (1980, 1981); the specific treatment as applied to the Yale code is given in Pinsonneault (1988).

We construct pre-main sequence (PMS) starting models as polytropes. For massive stars, an $n = 3$ polytrope works best. Starting models for rotation are based on observations for main sequence surface rotation rates (Fukuda 1982). Since the surface rotation should not change too much over the course of the CHB phase, it is reasonable to apply these rates to ZAMS models. Rigid rotation in the starting models is assumed.

3.10. MODEL PARAMETERS

The parameters used for these models were obtained from the recently published Yale standard solar model (Guenther *et. al.* 1992). The Anders-Grevesse mixture was used (Anders & Grevesse 1989), as is required for consistency if the OPAL opacities and Kurucz atmospheres are used. Rotation parameters were obtained in the same way, using observationally calibrated models of the rotating Sun. These parameters were supplied by Pinsonneault (1991). Mass loss is performed in the code using an empirically determined mass loss rate. Initially, the formulation of Waldron (1984) was chosen. Unfortunately, this gives mass loss rates that are much too high during the RSG phase, so it became necessary to match this parameterization with that of Reimers (1975).

4. Results of Evolutionary Calculations

4.1. THE EFFECTS OF ROTATION

Our results show that if rotational effects are included without the angular momentum redistribution, and the resulting mixing, there is very little effect on the evolution. The CHB and CHeB lifetimes are extended by approximately 1 per cent. The structure is virtually unchanged.

However, as can be seen in Fig. 1, the addition of rotational instabilities greatly affects the evolution of a 15 $M_\odot$ model. Initially, there is an increase in luminosity, and the main sequence extends further toward the red. The CHB lifetime is extended, as is the case for models including overshoot. The main sequence lifetime is now 8 per cent larger than for the nonrotating case. During the supergiant phase, however, the differences become larger. The rotating model undergoes several loops and for the most part is always significantly more luminous than its non-rotating counterpart. Unfortunately, none of these loops is long-lived, and the model ends its CHeB phase as a red supergiant. The inclusion of rotational instabilities extends the CHeB lifetime by 36 per cent. Rotational mixing also affects where the models spends its CHeB stages. Approximately a third of this phase is spent as a blue supergiant, with a log T_{eff} of approximately 4.1. After this, the star rapidly moves to the red side of the H–R diagram and spends the remaining time as a red supergiant with a log T_{eff} of about 3.5. This should decrease the blue-to-red supergiant ratio.

Obviously, substantial rotationally-induced mixing must be occurring to affect the evolution in such a manner. The additional mixing virtually eliminates all intermediate convection zones during the main sequence evolution. The hydrogen profile is smoothed out by rotational diffusion, while the non-rotating model has a step-like profile.

The internal transfer of angular momentum has important effects on the surface rotation velocity. Angular momentum is transferred from the core to the envelope, and the net result is a higher surface rotation velocity than for the case of no rotational instabilities. During the CHB phase of evolution, the additional angular momentum is sufficient to keep the outer portions of the model rotating at rates within the observed ones for the entire

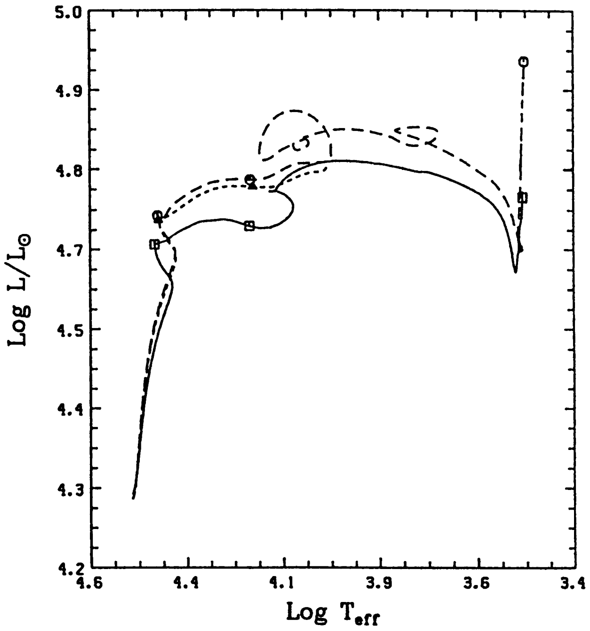

Fig. 1. Evolutionary track for a model without overshoot, mass loss or rotation (solid), rotation with instabilities, but no overshoot or mass loss (medium dashes), and with mass loss and rotation with instabilities, but no overshoot (short dashes).

main sequence stage. Unfortunately, the results are not as satisfactory for the supergiant phase of evolution. Here, the additional angular momentum causes the model to rotate at over twice the observed rates for approximately half of the CHeB lifetime. After this, the surface rotation drops well below the observable limits.

4.2. ROTATION AND MASS LOSS

The evolutionary track for a model with both rotation (including instabilities) and mass loss shows that mass loss has little effect on the evolution during the main sequence phase. The CHB lifetime is extended by 9 per cent, or 1 per cent more than the extension due to rotation alone. Mass loss has progressively greater effects during the supergiant phase. Overall, the luminosity of the entire sequence is decreased.

The structural evolution during the CHB stage shows that the addition of mass loss causes only slight changes in the structure during this phase of evolution. The rotation rate is largely unaffected during main sequence evolution . As the total mass and angular momentum lost increase, however, the effects become more significant. The addition of mass loss will at least give closer agreement with the observed surface rotation rates during CHeB evolution.

The addition of mass loss does not significantly improve the agreement of the rotating models with the observed TAMS. The main sequence phase still does not extend far enough into the red, suggesting that an additional mixing process may be necessary. One possibility would be a combination of rotation, mass loss and convective overshoot. It is also probable that the TAMS used here is too red (Stothers 1993). As in the previous case, however, the present models are merely suggestive. More massive sequences are needed before any definite conclusions may be made.

5. Conclusions and Suggestions for Future Work

Given the work that remains to be done, this is merely a preliminary study of the effects of rotation on the evolution of massive stars. However,there are also a number of valid conclusions which can be drawn.

1. We have confirmed that use of the Schwarzschild criterion will not produce extended blue loops during core-helium-burning evolution.
2. The current models need additional mixing beyond that produced by 0.1 H_p of convective overshoot, to agree with the shape and extent of the upper main sequence. Greater amounts of mass loss might cause sufficient main sequence extension, but would be unrealistic given the observed mass loss rates. Another possible explanation is that the TAMS used here is too red. As has been pointed out by Stothers (1993), Mermilliod & Maeder (1986) used a conversion between spectral type and temperature (Böhm-Vitense 1981) which is now known to be too cool.
3. The effects of rotational distortion are fairly small for a ZAMS surface rotation of about 200 km s^{-1}, starting with a rigidly rotating pre-main sequence model.
4. The effects of additional mixing due to rotational instabilities in some sense mirrors that of a small amount of convective overshoot. It does not induce an extended blue loop during the supergiant phase of evolution.
5. A combination of mass loss and rotation produces main sequence and supergiant surface rotation velocities which are in good agreement with observation.
6. Rotation and mass loss alone are insufficient to provide good agreement with the observed shape and extent of the upper main sequence.

This study opens the door for a great deal of future work. The evolutionary tracks must be extended further, more complex combinations of processes must be considered simultaneously, and different metallicities must be explored. This paper only reports the beginning of what promises to be a very exciting avenue of research.

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Discussion

Smith: Would the introduction of differential rotation due to pre-ZAMS contraction accelerate or decelerate the differential rotation you currently find in your post-ZAMS calculations?

Sofia: Differential rotation due to pre-ZAMS contraction would slightly enhance the amount of differential rotation in the MS and post-MS stages of evolution, and consequently, slightly enhance the effects of rotation.

Moss: Uniform rotation may not be the appropriate solution for a convective region if rotation causes anisotropy of turbulence and related angular momentum transport (cf. Rudiger 1989). Inter alia, this might influence the critical angular momentum distribution *throughout* the interior of low-mass stars after any Hayashi turbulence has disappeared.

Sofia: The existence of differential rotation in convective regions is not only possible, but actually is known (from helioseismology) to exist on the solar convective envelope. To begin with, the simplification was made as merely a conservative assumption. Subsequently, observations have shown that the departure from the assumption is very small, at least in the solar case.

Harmanec: Your models predict a decrease of equatorial rotational velocity during evolution away from the ZAMS to the TAMS. Can you envisage a reasonable change of your input physics which would lead to increase of rotation during the MS evolution?

The problem of excessive number of blue supergiants may have a simple solution: that many of them are MS objects with extended shells, i.e., Be stars. I have suggested this in various contexts several times previously.

Sofia: Since the moment of inertia increases moderately during the MS evolution, the only way you can get an increase of the surface velocity is if the star reaches the ZAMS with considerable differential rotation, which causes an outward transport of angular momentum during the MS evolution. Such a scenario cannot be dismissed. However, at the present time, we do not have any independent evidence in support of its existence.

Dziembowski: (1) You said that in your stellar evolution code you apply the Schwarzchild criterion for onset of convection in chemically inhomogeneous zones. Does your code mix everything (entropy, elements, angular momentum) if such zones are found unstable?

(2) I think that the best physical justification exists for mixing entropy and not elements because the instability is not of a dynamical type but rather vibrational.

Sofia: (1) We only mix the chemical elements. The entropy gradient is computed in the conventional (MLT) fashion, and because of the solid body rotation assumption, the specific angular momentum increases as r^2.

(2) The possibility of angular momentum transport without mixing due to oscillations is interesting and very real. Unfortunately we have not yet explored how to incorporate it in our evolution codes.

Henrichs: Which opacities did you use for your calculations?

Sofia: The OPAL (Lawrence Livermore) opacities.

Marlborough: Do your calculations with mass loss include the entropy loss to the system required to transport the matter to infinity?

Sofia: Yes.

Heap: I wonder if all the observed problems you describe are real, in particular, the shape of the upper main sequence and the lack of a post-MS gap. The problems may be on the observational side: in deriving the surface parameters (T_eff, $\log g$) from the spectrum via NLTE models with winds. Until we can do that with reliability, we can't really say where the ZAMS is or where the post-MS is on the HR diagram.

Sofia: It is always tempting for the theorist to accept the observer's results without a great deal of questioning. Indeed, it might be inappropriate for us to suggest that our failure to solve a given perceived problem is due to faulty observations rather than shortcomings of our models. Of course, our continued inability to explain a given observation may lead us to begin to doubt its existence. However, only other (better) observations can disprove it.