# SELECTED TOPICS ON THE EVOLUTION OF LOW AND INTERMEDIATE MASS STARS

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ABSTRACT

Among the many aspects of the evolution of low and intermediate mass stars, two representative topics are selected for this review: the question why stars become red giants, and the problem of the age determination of galactic globular clusters. Concerning the first topic, it is shown that this happens because of a thermal instability taking place in the stellar envelopes, and the physical nature of this instability is identified. Several ramifications of these findings are then briefly mentioned. Concerning the second topic, the Oosterhoff-Sandage effect is briefly described, together with its implications for the age estimates and the problems rised by its interpretation in terms of evolutionary models. In this connection, it is suggested that an enhanced overall abundance of the elements CNO and Ne may solve these problems, although further evolutionary calculations are required before reaching firm conclusions. It is also amphasized for both topics the important role played by the metal contribution to the radiative opacity at middle temperatures.

## 1. INTRODUCTION

The evolution of low and intermediate mass stars offers a number of exciting aspects and problems. First comes the question of understanding *everything* happening to such stars, from their pre-main sequence stage to their final fate, of determining the diversity of their evolutionary behaviours depending on initial mass and composition, and then of comparing theoretical predictions with observations. This includes the problem of assessing the *accuracy* and *adequacy* of current canonical models in all evolutionary stages, and therefore of identifying the role, if any, played by those physical processes which are left out in the canonical approximation (like rotation, magnetic fields, mixings of non-convective origin, effects of the stellar environment, etc.). The accuracy of cano-



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nical models clearly depends upon the accuracy of the input physics used in constructing such models, which includes opacity, nuclear reaction rates, neutrino losses, equation of state, etc. It should be emphasized that the question of the accuracy of the models must logically be asked *before* that of the adequacy of the canonical approximations. Otherwise, when a discrepancy between model predictions and observations is perceived, one would not know whether to blame the insufficient accuracy of the input physics, or the inadequacy of the canonical assumptions, thus producing confusing answers to ill-posed questions.

The second major aspect of stellar evolution theory (in particular for what concerns low and intermediate mass stars) refers to its applications in the broader astrophysical context, when theoretical evolutionary models are used to infer several global properties of various stellar aggregates. For example, age determinations, star formation histories, chemical and spectral evolution of galaxies, and the like. Certainly, before using the models it would be preferable to have first successfully checked their reliability. But, how to resist the temptation of employing such a powerful tool, when astronomers are challenged by so many fascinating questions? After all, the use of the tool refines the tool itself.

As mentioned at the beginning, the number of problems posed by/to the theory of stellar evolution is virtually illimitate. Therefore, for this review I have singled out just a few topics, chosed as interesting examples illustrating some of the situations mentioned above. These (and other) aspects and applications of stellar evolution theory are also discussed in a recent series of reviews by Icko Iben and myself (Iben & Renzini 1983, 1984; Renzini & Iben 1984).

Section 2 of this paper deals with an old question: *why do stars become red giants?* In my opinion this question did not yet receive a satisfactory answer in the literature. I will then present my own explanation for this phenomenon, and will also try to show that its better understanding may effectively help in solving several other open problems. Finally, Section 3 deals with the Oosterhoff-Sandage effect in galactic globular clusters, and with the question of determining the age of these oldest stellar aggregates. In closing this section the problems posed by the presence of unexpected gaps on the main branches of some cluster C-M diagrams will be briefly discussed. In both sections reference will be often made to the "metal opacity at middle temperatures". With this expression one means the contribution provided to the total radiative opacity by the elements heavier than helium, at temperatures between roughly half a million and five million Kelvins. Indeed, in this temperature range, bound-bound and bound-free transitions in the few last ionization stages of these elements provide an important contribution to the total radiative opacity (cf. Renzini 1977, Fig. 2.3).

## 2. ON WHY STARS BECOME RED GIANTS

Color-magnitude diagrams (for instance for stars in the solar neighborhood) show many stars near the main sequence, many red giants, but very few stars at intermediate temperatures. This is the well known "Hertzsprung Gap". Theoretical models for stars roughly more massive than 1.2 $M_\odot$, and near-solar metallicity, experience a rapid expansion shortly after the beginning of hydrogen-shell burning, and on a short (thermal) timescale reach red giant dimensions. Following their arrival on the Hayashi track, thermal balance is restored and the evolution proceeds on a nuclear timescale. So, there is excellent agreement between theory and observations: the existence of the Hertzsprung Gap is clearly reproduced. But, why models (stars) behave as they do?

### 2.1 Three illuminating cases

The answer to this question is rather complex, and requires a number of preliminary steps. I shall first discuss three cases which will help a clearer formulation of the problem.

*Case 1*: Models with (Y, Z) = (0.28, 0.01) and M = 3 to 7 $M_\odot$. Let us consider the evolutionary tracks shown in Fig. 1b in Becker et al (1977). These three models behave in a very similar way: following central hydrogen exhaustion (point 2), the surface luminosity increases until a relative maximum is reached, then the luminosity drops while the envelope is rapidly expanding (on a thermal timescale). The drop in surface luminosity continues until the model approaches the Hayashi track, envelope convection penetrates deep into the interior, and a relative minimum in luminosity is reached (point 4). While models evolve from the relative maximum to the relative minimum in luminosity, their envelopes depart dramatically from thermal equilibrium: a relevant fraction of the luminosity produced in the interior being absorbed in the envelope to sustain envelope expansion (cf. Iben 1965). Following point 4 thermal equilibrium is restored. One can conclude that these models become red giants because of a thermal runaway taking place in the envelope. Note that qualitatively similar runaways take also place following the central helium exhaustion. Models of smaller mass (but more massive than $\sim 1$ $M_\odot$) also experience the thermal runaway shortly after the central hydrogen exhaustion (cf. Mengel et al 1979).

*Case 2*: Models with (Y, Z) = (0.28, 0.001) and M = 3 to 7 $M_\odot$. Fig. 1a in Becker et al (1977) shows what happens to the evolutionary tracks when the metal abundance is decreased by a factor of 10, compared to the previous case. The 3 $M_\odot$ model behaves qualitatively as in the previous case, i.e. it suffers the envelope thermal runaway, and ignites helium at the center as a red giant. In the 5 $M_\odot$ model, the thermal runaway has just started

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when it is aborted by the central He ignition (point 5). The model experiences the runaway only following the central He exhaustion. This latter phase is qualitatively similar to the corresponding situation at higher Z. Finally, in the 7 $M_{\odot}$ model the thermal runaway does not even begin: one can say that the central He ignition anticipates (and then prevents) the onset of the thermal runaway. As in the 5 $M_{\odot}$ model, a thermal runaway develops only after the central He exhaustion. From the comparison of these two cases one can conclude that the abundance of heavy elements plays a crucial role in determining whether model stars become red giants, and in which evolutionary stage they do so. More specifically, the abundance parameter Z affects the evolution of stars through the rate of hydrogen burning (CNO cycle), and through the radiative opacity in the middle temperature regime (cf. Section 1). Since the thermal runaway takes place in the envelope, one can tentatively identify in the opacity (rather than in the nuclear burning) the leading cause for the different behaviour of the models at different Z's. This conclusion will be reinforced by the examination of the next case.

*Case 3*: Models of massive stars with unconventional middle temperature opacities. Stothers & Chin (1977), Bertelli et al (1984) and Greggio (1984) have computed evolutionary sequences for massive stars, using unconventional (higher) opacities at middle temperatures. In their models more massive than $\sim 20\ M_{\odot}$, the envelope thermal runaway starts while hydrogen is still being burned at the center, thus producing core hydrogen burning red giants. These numerical experiments are physically interesting, independently of the question whether such unconventional opacities are correct or not.

## 2.2 First inferences

From the three cases discussed above we can derive the following preliminary conclusions:

1) At least in these cases, stars become red giants because of the onset of a thermal instability in the envelope (runaway envelope expansion).

2) Reaching the so-called Schönberg-Chandrasekhar limit is neither a necessary nor a sufficient condition for stars to become red giants. In fact, in Case 2 (5 and 7 $M_{\odot}$ models) the S-C limit is reached but models do not become red giants (if not after core He exhaustion), and in Case 3 models become red giants well before reaching the S-C limit. The traditional misconception, associating the approach to the S-C limit with the tendency of the models to expand to red giant dimensions, arises just from Case 1 (near-solar Z), when it happens that these two events take place rather close in time to each other. Indeed, the Schönberg and Chandrasekhar theorem, as originally stated (Schönberg & Chandrasekhar 1942), shows only the propensity of the core to contract rapidly, not that of

the envelope to expand (Eggleton & Faulkner 1981).
3) The metal opacity at middle temperatures must play a crucial role in determining the onset of the envelope thermal instability. Indeed, by varying such opacity (either artificially or varying Z) the thermal instability can be prevented (cf. Case 2 with Case 1), or triggered at an earlier evolutionary stage (cf. Case 3 with Case 1). Therefore:
4) The first occurrence of the thermal instability cannot be unambiguously associated with any particular evolutionary stage, i.e. with any particular structure of the core. It may happen during the stage of core hydrogen burning (Case 3), or during the early shell hydrogen burning stage (Case 1), or delayed to the double-shell burning stage (Case 2, 7 $M_{\odot}$ model). The lesson to draw from these considerations is that, if we want to understand why stars become red giants, our attention must primarily be focused on the behaviour and properties of star envelopes, rather than to what happens to stellar cores, i.e. we must identify the physical origin of these thermal instabilities in stellar envelopes.

## 2.3 The physical nature of the thermal instability

The main factor driving the evolution of stars is certainly represented by the continuous nuclear transmutations taking place in the deep interior. Indeed, such transmutations affect the mean molecular weight (and then the equation of state), the opacity, and the rate of the nuclear reactions, thus causing the stellar structure to continuously readjust in order to maintain a quasi-static configuration. In all canonical models (e.g. those discussed above) these transmutations are confined to the deep interior: i.e. they do not extend to the envelope. Therefore, the envelope *follows* the evolution of the core: it is just called to readjust itself to the changing structure of the core, and primarily to the changing luminosity being provided by the core. There are not so many possible readjustements: basically, the envelope can only either contract or expand. The question then becomes: how does a stellar envelope react to the changing luminosity being provided by the core?

To answer this question let us start by considering one of the basic equations of stellar structure:

$$L_r = 4\pi r^2 \times F_r = 4\pi r^2 \, (4acT^3/3\kappa\rho)(dT/dr) \qquad (1)$$

which simply says that the radiative luminosity (at any given mass coordinate $M_r$) is given by the radiative flux $F_r$ times the area through which this energy flows. All the symbols have here their traditional meaning. Let us now focus our attention on a radiative layer located at a given mass coordinate $M_r$, and suppose to expand this layer in such a way as to maintain hydrostatic and radiative equilibrium. Then the area $4\pi r^2$ increases and the flux $F_r$ decreases, but we don't a priori know whether their product (the transmitted luminosity $L_r$) will increase or decrease. This will depend on several details. However, one can differentiate Eq. (1) to get (to first order in $\delta\log r$):

$$\frac{\delta\log L_r}{\delta\log r} = W(M_r) \simeq 4(1 - A) + (4 - A)\alpha - A\beta \qquad (2)$$

where $A = -(\delta\log T/\delta\log r)$, $\alpha = (\partial\log\kappa/\partial\log\rho)_T$, and $\beta = -(\partial\log\kappa/\partial\log T)_\rho$. The function $W(M_r)$ describes how the transmitted luminosity $L_r$ varies in response to a local infinitesimal expansion or contraction $\delta\log r$. While the utility of Eq. (2) is somewhat limited by the fact that the quantity A cannot be precisely determined from elementary arguments, nevertheless this relation clearly shows that the size and sign of $W(M_r)$ are crucially dependent on the opacity derivatives $\alpha$ and $\beta$. This is further strengthened by the fact that (in actual evolutionary models) $A(M_r)$ is close to unity (to within $\pm\sim 10\%$) over most of the stellar envelope, which is equivalent to saying that evolutionary changes in the envelope are *locally* virial to a fairly good approximation, as noted a long time ago by Iben (1965). So, the three terms at the r.h.s. of Eq. (2) are of comparable magnitude. Another, though equivalent way of looking at Eq. (2) is to interpret the differentials $\delta\log L_r$ and $\delta\log r$ as the actual changes (at any given $M_r$) between two consecutive evolutionary models, i.e. $W(M_r)$ can also be defined as:

$$W(M_r) = (d\log L_r/dt)\cdot(d\log r/dt)^{-1} \qquad (3)$$

where the evolutionary time derivatives are evaluated at any given $M_r$. In this way, the function $W(M_r)$ can easily be evaluated for any given model belonging to an evolutionary sequence.

After these premises, let us see how the function $W(M_r)$ can be used to clarify the nature of the envelope thermal instability. A $W(M_r) > 0$ implies that, following an infinitesimal expansion, the radiative flux drops by an amont which is less than the corresponding increase in the surface area, and then the transmitted radiative luminosity increases. The opposite holds when $W(M_r) < 0$, i.e. a local expansion implies a decrease in the transmitted luminosity. At this point one can easily realize that a thermal instability can appear when $W(M_r) < 0$. In fact, in this case an envelope expansion implies a decrease in the transmitted luminosity, energy is then trapped in the envelope causing further expansion and a further decrease of the transmitted luminosity: the result is clearly a runaway expansion of the envelope. Such a runaway expansion can be halted only if for some reason $W(M_r)$ can return to positive values.

The inverse function:

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$$W(M_r)^{-1} = (\delta \log r/\delta \log L_r) \quad (4)$$

has also an interesting physical meaning. Indeed, it describes how an envelope layer reacts to a variation of the luminosity impinging on the inner side of that layer, i.e. an increase in the luminosity $\delta\log L_r$ produces an expansion $\delta\log r$ given by $W(M_r)^{-1}\delta\log L_r$.

Model calculations (Renzini & Chieffi 1984) show that in the envelope of core hydrogen burning models (with standard opacities) $W(M_r)$ is a *positive*, slowly radially decreasing function of $M_r$. Since the luminosity being provided by the burning core is secularly increasing during this phase, Eq.s (2) and (4) imply that the envelope must expand, and with a rate which increases outwards (since $W(M_r)$ decreases from the core to the surface). The fact that $W(M_r) > 0$ also ensures that thermal equilibrium can be maintained in the envelope: the increasing luminosity of the core forces the envelope to expand, but by such an expansion the luminosity transmitted by the envelope also increases, and near equality can be maintained between the core luminosity and the luminosity transmitted by the envelope and radiated away from the surface. One can conclude that envelopes with $W(M_r) > 0$ are thermally stable.

However, this favourable situation does not necessarily apply to all evolutionary conditions. During the main sequence phase most of the envelope is at relatively high temperatures ($T > \sim 5\ 10^6$ K), where electron scattering dominates and the opacity derivatives $\alpha$ and $\beta$ are vanishingly small. However, as the star expands, an increasing fraction of its envelope cools below $\sim 5\ 10^6$ K, the metal opacity increases, and $\alpha$ and $\beta$ tend towards the Kramers values 1 and 3.5, respectively. Therefore, for every $M_r$ in the envelope, $W(M_r)$ secularly decreases as the evolution proceeds. This trend is maintained during the early hydrogen-shell burning phase, until (for models like those of Case 1 in Section 2.1) $W(M_r)$ vanishes, *at first near the surface*, and then becomes negative, with the point where $W(M_r) = 0$ moving *rapidly inward* in mass (Renzini & Chieffi 1984). This coincides with the onset of the thermal runaway in the envelope. Indeed, what happens when $W(M_r) \to 0$? From Eq. (4) one has that $\delta\log r/\delta\log L_r \to \infty$, i.e. a small increase in the luminosity emanating from the central regions triggers a catastrophic expansion of the envelope. Moreover, further expansion leads to negative values of $W(M_r)$, an event already discussed above, together with its implications. The runaway will be quenched later, by the inward penetration of convection, when then Eq.s (1) and (2) lose their validity. One can really say that convection saves the envelope from being dynamically ejected!

It is most important to emphasize again that $W(M_r)$ first vanishes near the surface. This clarifies several interesting aspects of the problem: that the thermal instability arises in the envelope itself (not in

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the deep interior), that the behaviour of the core works just as a trigger for this instability, that the opacity derivatives $\alpha$ and $\beta$ play a crucial role in determining the onset of the instability, which explain the different behaviour of the models in the Cases 1, 2 and 3 described in Section 2.1.

## 2.4 Consequences

This identification of the physical reason "why stars become red giants" has a variety of astrophysically interesting ramifications. Some of these implications are here briefly mentioned, acknowledging that many details need to be worked out before achieving a deeper quantitative understanding of the mentiond phenomena, and even a more rigorous formulation of some of the following statements themselves. More on these topics will be presented in a future paper (Renzini & Chieffi 1984).

1) Stars (more massive than $\sim$1 $M_\odot$) become red giants because of a thermal instability in the envelope, whose onset is primarily controlled by the behaviour of the opacity at middle temperatures, i.e. by $\alpha$ and $\beta$.

2) The cool temperature boundary of the so-called main sequence band coincides with the locus where $W(M_r)$ vanishes at the surface of evolutionary models. Therefore, the width of this band is expected to be (highly?) sensitive to the metal contribution to the opacity at middle temperatures, i.e. again to $\alpha$ and $\beta$. Ultimately, the thermal instability can be described as a runaway partial recombination of the heavy ions in the envelope.

3) In the models lying in the so-called Hertzsprung Gap the function $W(M_r)$ is negative and/or close to zero. When $W(M_r)^{-1}$ is very large, any small change in the luminosity emitted by the core can lead to large changes in the stellar radius (effective temperature). This implies that when $W(M_r) \simeq 0$ small changes in the input physics may lead to large changes in the morphology of the tracks. One should always bear this in mind when comparing theoretical sequences with observations: in other words, the *accuracy* of theoretical effective temperatures is rather low in this region of the HR diagram.

4) The thermal instabilities associated with the beginning and termination of the core helium burning loops (cf. Fig. 1 in Becker et al 1977) have essentially the same physical origin, and can be explained in terms of the behaviour of the function $W(M_r)$. In particular, the runaway envelope *contraction* experienced by some models during the core helium burning stage is another manifestation of the same type of envelope thermal instability, here working in the *reverse* direction.

5) From points 3) and 4) it follows that the theoretically predicted distributions of the periods of Cepheids (cf. Becker et al 1977) must be very sensitive to the metal opacity (through $\alpha$ and $\beta$), and to small changes in the input physics as well.

6) From points 2), 3) and 4) it follows that theoretical blue to red ratios for the number of massive stars (a popular topic at this meeting) is expected to be very sensitive to $\alpha$ and $\beta$ and to small changes in the input physics as well. This suggests extreme caution when comparing such ratios with the observations.

7) Finally, it is of great importance to realize that in models lying in the Hertzsprung Gap most of the material in the stellar envelope is at temperatures between 1 and 5 million K. In several cases, up to 70 % of the stellar mass has temperatures in this range. Still, practically all existing evolutionary models have been constructed using tables giving the opacity at only three temperatures in this range, just where $\alpha$ and $\beta$ suffer the largest and most significant changes. Even worse, interpolation schemes which fit to these tables may give large discontinuities in $\alpha$ and $\beta$ at tabular $\rho$ and T values, thus adding further uncertainties to the model radii. The use of analytical approximations has at least the advantage of providing continuous derivatives of the opacity.

In conclusion, two are the main lessons to be drawn from these considerations: a) theoretical stellar radii (effective temperatures) cannot be very accurate (in several circumstances), and b) the full understanding of why models behave as they do is essential for their fruitful and correct use in the comparison with the observations. In other words, a simplistic use of the models may produce astrophysical nonsense.

## 3. THE OOSTERHOFF-SANDAGE EFFECT AND THE AGE OF GALACTIC GLOBULAR CLUSTERS

The determination of the age and helium abundance of galactic globular clusters is certainly the topic in which the connection between stellar evolution theory and cosmology is most straightforward. Deriving cluster ages from a comparison of observational C-M arrays with theoretical models (isochrones) is apparently an easy game. However, this is not the case if one wants to derive ages with the accuracy requested in order to be interesting for cosmology. There are many methods which have been used to determine cluster ages. Each of them requires many intermediate steps to get the age starting from the "brute" C-M diagram of a cluster. But all of them ultimately rely on theoretically established relationships between the characteristics of the main sequence turnoff and age.

The method which has been most widely adopted is the isochrone-fitting technique. However, I think that this method has so many disadvantages that its further use should be strongly discouraged. The main flaw is that, when fitting theoretical and observed C-M *shapes*, one gives at least equal weight to colors (temperatures) as does to luminosities. But colors (temperatures), in the relevant region from the turnoff to the red giant branch, are strongly affected by the uncertain treatment of the envelope convection (cf. Iben & Rood 1970a), and are much more sensitive to metallicity than they are to age (Iben & Renzini 1984). While at least part of the first difficulty can be removed by a proper choice of the mixing-length parameter $\ell/H$ (as done recently by Vandenberg 1983), there remains the metallicity problem: a variation in Fe/H by 0.3 dex (a rather optimistic estimate of current uncertainties) affects the turnoff temperature by the same amount produced by an age variation up to $\sim$4 Gyr (1 Gyr = $10^9$ years). Moreover, uncertainties in reddening corrections, color-temperature transformations, bolometric corrections, and quite possible systematic photometric errors at the faint end of the sequences, further complicate the game. For these reasons (and others as well) one should always prefer a cluster dating method in which the use of stellar effective temperatures is reduced to a minimum.

A method with these characteristics was indeed developed in the late 60's (e.g. Iben & Rood 1970b, see also Iben 1974, Renzini 1977, Iben & Renzini 1984). This method relies on a theoretically established relationship between the luminosity of horizontal branch (HB) stars (at a given temperature), and the composition (Y, Z) of the main sequence progenitors of these stars. It further relies on a theoretically established relationship between the luminosity at cluster turnoff and age (which also involves Y and Z). From photometric observations one can obtain the difference in luminosity between stars at the cluster turnoff and HB stars at the *same color* (same temperature). From spectroscopic observations one can estimate Z. Estimating Y by various methods then allows one to solve for age. Note that in this case stars with the same color are compared, and so uncertainties in convection treatment, reddening, color-temperature transformations, bolometric corrections and the like are largely avoided. The sensitivity to Z is also significantly reduced (Iben & Renzini 1984).

However, while avoiding the mentioned uncertainties, this method requires the luminosity of HB stars to be very accurately reproduced by the models. Indeed, an error of one tenth of a magnitude in the HB luminosity level translates into an error of 1-2 Gyr in the derived cluster age. Independent checks of the accuracy of HB models are thus necessary for assessing the accuracy of the derived ages. Fortunately enough, RR Lyrae variables are frequently present in large numbers in globular clusters thus allowing a consistency check between pulsation theory and evolutio-

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nary models, and intimately connecting to each other the study of RR Lyrae variables and the determination of cluster ages (cf. Sandage 1970, Iben 1971, 1974). This rather intricated connection is the subject of the following discussion.

## 3.1 Three empirical facts

It is worth starting this discussion by recalling three crucial observational facts.

1) *The Oosterhoff effect.* It has been known for a long time that the average period of globular cluster RR Lyraes pulsating in the fundamental mode ($<P_{ab}>$) shows a remarkable bimodal distribution (Oosterhoff 1939, Van Agt & Oosterhoff 1959, cf. also Cacciari & Renzini 1976, for a more recent compilation of the data). Clusters with RR Lyraes are either of Oo. type I ($<P_{ab}> \simeq 0^{d}.55$, prototype M3), or of Oo. type II ($<P_{ab}> \simeq 0^{d}.65$, prototype M15). Obviously, an Oosterhoff type cannot be assigned to those clusters with too few RR Lyraes (if any). However, these clusters can be devided into two further groups: those lacking RR Lyraes either because their HB is too blue (BHB clusters, prototype M13), or too red (RHB clusters, prototype 47 Tuc). In this way, galactic globular clusters are classified into four natural groups, according to their HB morphology, and the properties of their RR Lyraes.

2) *The Sandage effect.* Sandage (1982) has recently discovered that the period of RR Lyraes at a given effective temperature is a decreasing function of the metallicity of the parent cluster, i.e.:

$$\Delta \log P \simeq -0.116\ \Delta[\mathrm{Fe/H}]\,. \tag{5}$$

Note the different sign convention with respect to Eq. (2) in Sandage (1982). The reality of this effect is convincingly demonstrated by the presence of a similar *period shift* when comparing variables with the same amplitude or with the same light curve shape. Moreover, effectively the same period shift is found by Lub (1977) for *field* RR Lyraes in the fundamental mode, and by Kemper (1982) for overtone variables.

By coupling the *empirical* relation (5) with the theoretical Period-Luminosity-Mass-Temperature relation for RR Lyrae stars, Sandage then derives:

$$\Delta M_{bol}^{RR} \simeq 0.35\ \Delta[\mathrm{Fe/H}] \tag{6}$$

for the variation of the bolometric magnitude of the RR Lyraes with the metallicity of the parent cluster. As we shall see later, the relations (5) and (6) are of great importance for the determination of the age of globular clusters.

3) *The HB morphology*. Having defined four natural groups of clusters, it is interesting to examine the average metallicity of the clusters in each group. This was accomplished by Renzini (1981, 1983) using the metallicities given by Zinn (1980), with the somewhat surprising result that the average metallicity <[Fe/H]> in each group of clusters increases along the sequence OoII, BHB, OoI, RHB, assuming respectively the values -2.1, -1.7, -1.5, and ∿-0.8. The typical overlap in [Fe/H] between one group and the next is 0.2-0.3 dex, quite consistent with the overlap being (mostly) due to observational errors. We see, then, that with increasing metallicity, the HB first moves to the blue (from OoII to BHB clusters), then this trend is reversed with the HB entering again the RR Lyrae instability strip (from BHB to OoI clusters), and eventually exiting from the opposite side of this strip (from OoI to RHB clusters). Renzini (1981, 1983) and Iben & Renzini (1984) briefly discuss the possible origin of this non-monotonic behaviour, and emphasize the relevance of this empirical fact for understanding the old question of the so-called "Second Parameter", (apparently?) required to explain the poor correlation between HB morphology and cluster metallicities.

What matters here is that this non-monotonic behaviour can naturally account for the heretofore puzzling dichotomy in $\langle P_{ab} \rangle$ between the two Oosterhoff groups. In fact, the existence of a gap in metallicity between OoII and OoI clusters (around [Fe/H] = -1.7, the range occupied by BHB clusters), coupled with Sandage's period shift effect, automatically ensures the existence of the discontinuity in $\langle P_{ab} \rangle$ between the two groups. More precisely, inserting $\Delta$[Fe/H] = 0.6 into Eq. (5) one derives $\Delta \log P = -0.07$, which is just the difference in log $\langle P_{ab} \rangle$ between the two groups! This merely empirical explanation of the Oosterhoff effect is still valid even if the spectroscopic metallicities of Pilachowski et al (1983) are adopted, since the [Fe/H] gap between OoII and OoI clusters remains almost unchanged (Iben & Renzini 1984).

## 3.2 A theoretical embarrassment

What remains to be understood in terms of evolutionary models is the origin of the period shift effect, Eq. (5). Indeed, current HB models (e.g. Sweigart & Gross 1976) predict a negligible period shift, $\Delta \log P \simeq -0.02\, \Delta$[Fe/H] , if all clusters have essentially the same helium abundance (Sandage 1982, Renzini 1983, Iben & Renzini 1984). In an attempt to reproduce Eq. (5) with current models, Sandage was then forced to suggest an anticorrelation of the helium abundance with metallicity, i.e. $\Delta Y \simeq -0.07\, \Delta$[Fe/H] , with metal poor clusters resulting richer in helium. As acknowledged by Sandage, "the sense is against intuition", then prompting a search for alternative interpretations.

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An obvious limitation of Sweigart & Gross HB sequences is that they are available for only one vale of [CNO/Fe] , namely [CNO/Fe] = 0, while there is clearly no a priori guarantee that elemental proportions in the halo should be the same as in the sun. However, adopting [CNO/Fe] = -a[Fe/H], with a = const., does not solve the problem, as noted by Renzini (1983) on the basis of the models of Rood & Seitzer (1981): the period shift remains unconfortably small.

This discrepancy between the predicted and observed period shifts indicates that, for some reason, current HB models are not sensitive enough to [Fe/H]. Therefore, let us see in more detail how the period shift is related to [Fe/H] when pulsation and evolution theories are jointly used. Indeed, from pulsation theory one gets, at fixed temperature:

$$\Delta\log P = 0.84\ \Delta\log L - 0.68\ \Delta\log M \tag{7}$$

where L and M are the luminosity and mass of the RR Lyraes. From evolution theory, we can now relate Δlog L and Δlog M to a variation of metal abundance Z:

$$\Delta\log L = \{(\partial\log L/\partial M_c)_{YZ}(\partial M_c/\partial\log Z)_Y + (\partial\log L/\partial\log Z)_{M_c Y}\}\ \Delta\log Z \tag{8}$$

$$\Delta\log M = \{(\partial\log M/\partial M_c)_{YZ}(\partial M_c/\partial\log Z)_Y + (\partial\log M/\partial\log Z)_{M_c Y}\}\ \Delta\log Z \tag{9}$$

where $M_c$ is the core mass at the helium flash, and all the partial derivatives can be evaluated from evolutionary sequences. For example, when the partial derivatives are obtained from the models of Sweigart & Gross (1976, 1978), then (8) and (9) inserted into (7) give the too small period shift which has been reported above.

Therefore, the suspicion is that some of these derivatives may be in error, because of possible inaccuracies in the input physics, or for some other reason. Renzini, Sweigart & Tornambè (1983) have then undertaken a project aimed at checking these derivatives and their dependence on variations in the relevant input physics. Some preliminary results of this study are now reported here.

We first concentrated on the core mass at the helium flash. The value of $M_c$ is known to depend on the neutrino losses from the core that models experience while ascending the red giant branch (e.g. Sweigart & Gross 1978). However, these sequences were computed using a rate of neutrino losses for the plasma process (the only relevant process in this case) which does not include the contribution of neutral-current interactions. In order to take this into account we have then computed several red giant branch sequences, for various values of Z, and increasing the neutrino losses by a factor $F_\nu$ over the rate of Beaudet et al (1967), which does not include the effect of neutral currents. The result is:

$$(\partial M_c/\partial\log Z)_Y \simeq -0.009 - 0.004\ (F_\nu - 1). \tag{10}$$

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Hence, by increasing the neutrino losses the metallicity dependence of $M_c$ increases, and this goes in the desired direction to produce a period shift. However, according to Ramadurai (1976) a value $F_\nu \simeq 1.5$ is appropriate for taking neutral currents into account, and when inserting this value into (10) the resulting change is rather modest and largely insufficient to produce a period shift effect of the size demanded by (5).

Therefore, we next considered HB models, and in particular the third derivatives appearing in Eq.s (8) and (9). Following an early suggestion (Renzini 1983), we have computed zero age HB sequences with an artificially increased metal opacity at middle temperatures. (Note that only the metal opacity is increased, not the other contributions!) By increasing the metal opacity the luminosity of a HB model of given mass, core mass, and composition is not greatly affected, it decreases just a little. But the effective temperature is considerably decreased, and the net result is a shift of the zero age HB locus to lower temperatures (for a given luminosity) and to lower luminosities (for a given temperature). The mass of the models within the instability strip is also considerably decreased. The size of these effects increases with metallicity, and then, as expected, the size of the period shift is sensitive to the adopted opacity. However, the required increase over the Los Alamos metal opacities is at least by a factor of 5, if relation (5) is to be reproduced. This increase may appear as unrealistically large, i.e. it is unlikely that Los Alamos metal opacities are in error by such a large factor. Fortunately, there is another, more attractive way of achieving essentially the same result.

Indeed, the metal opacity at middle temperatures is mostly contributed by the elements CNO and Ne. Therefore, an increase in the abundance of these elements (relative to iron) will produce an increase in the metal opacity by nearly the same factor, and then in the size of the period shift. According to the previous numerical experiment, we estimate that a period shift of the correct size should be obtained by adopting [CNONe/Fe] $\simeq 0.7$. However, more work is needed to confirm this suggestion: first, new opacity tables with enhanced CNO and Ne abundances must be constructed, and, second, HB models using these tables have to be computed.

Concerning opacity tables, still a few considerations are in order. The total opacity at middle temperatures is highly non-linear with respect to abundances. Below Z(total) $\simeq 10^{-3}$ other opacity sources dominate, and variations in metallicity give small changes in the total opacity. The contrary happens above Z(total) $\simeq 10^{-3}$, when the metals (whichever their relative proportions) dominate the total opacity. Therefore, any trend established for Z(total) $< \sim 10^{-3}$ cannot be safely extrapolated at higher metallicities!

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It is also worth emphasizing that, in order to get the proper period shift, [CNONe/Fe] should be both *positive* and nearly *constant* in passing from OoII to OoI clusters, i.e. for [Fe/H] < ∿-1.3. Indeed, as mentioned before, a decreasing trend in [CNO/Fe] with increasing [Fe/H] does not solve the problem. Is all this reasonable? There is actually some observational support for this being the case: for instance, Sneden et al (1979) find [O/Fe] ≃ 0.5 for -2.3 ≲ [Fe/H] ≲ -0.5 in a sample of field metal poor dwarfs (see also Barbuy 1983, Pilachowski et al 1983, and references therein). Also from a theoretical point of view one may expect an overabundance of at least O and Ne in halo stars and clusters, since these elements are synthetized in short living massive stars, while an important fraction of Fe may come from type I supernovae having long-living progenitors. One can tentatively conclude that the [CNONe/Fe] ratio offers a viable possibility for explaining the origin of Sandage's period shift effect, although further theoretical and observational studies are required before reaching a firm settlement of the issue.

## 3.3 The age of globular clusters

Let us first consider the question of the age spread within the globular cluster family. From theoretical isochrones (e.g. Ciardullo & Demarque 1977, Vandenberg 1983) it follows that, at *fixed age*, [CNONe/Fe], and Y, the luminosity of the main sequence turnoff decreases with increasing metallicity, following the relation:

$$\Delta M_{bol}^{TO} \simeq 0.37\ \Delta[Fe/H]. \qquad (11)$$

From the near identity of the numerical coefficients in (6) and (11), it follows that the luminosity difference between the HB and the turnoff, $\Delta M_{TO}^{RR}(bol) = M_{bol}^{TO} - M_{bol}^{RR}$, should be almost independent of metallicity, if all globular clusters are coeval and have nearly the same Y and [CNONe/Fe] (Sandage 1982). Since observationally $\Delta M_{TO}^{RR}(bol)$ is actually constant to within the observational errors ($= 3^m.4 \pm 0.2$), one can infer that clusters are indeed coeval (Sandage 1982). Obviously, this statement applies to the clusters for which a photometrically accurate C-M array is presently available, and will have certainly to be revised if, in the future, other clusters will be found to have significantly different values of $\Delta M_{TO}^{RR}(bol)$. However, the current uncertainty in this quantity (±0.2 mag) implies an uncertainty of about 3.5 Gyr in each age determination, and therefore age differences of this size could not have been detected, although it is somewhat encouraging that no trend whatsoever is apparent in $\Delta M_{TO}^{RR}(bol)$ vs metallicity (cf. Sandage 1982).

It is certainly of great astrophysical interest to reduce this uncertainty. I think that this can only be achieved by using linear detec-

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tors (e.G. CCD's) and observing as many clusters as possible. Further photographic photometry of cluster turnoffs should rather be discouraged, since with this technique the turnoff luminosity can hardly be determined with an accuracy any better than the current value (±∿0.2 mag), and since linear detectors are already widely available.

Let us now turn to the question of the absolute age of the clusters. To get the age one needs a *zero-point* for the luminosity of the HB. Sandage proceeds on empirical grounds, adopting $M_{bol}^{RR} = 0.8$ for the RR Lyraes in the cluster M3, and then derives an average age for the eight studied clusters of 17±2 Gyr, the age of each individual cluster being uncertain by ±3.5 Gyr. Iben & Renzini (1984) proceed on theoretical grounds, and argue that HB models of very low metallicity are expected to be quantitatively more reliable, since in these models uncertainties in opacity and [CNONe/Fe] have smaller effects. Therefore, they adopt low metallicity HB models (Z = 0.0001) as providing the zero point for the HB luminosity, and further adopt Y = 0.23, as recently obtained by Buzzoni et al (1983) for a sample of 15 well studied clusters. This gives an age of 16±3.5 Gyr for the cluster M92, and near equality for the age of the other clusters follows from the argument given above, i.e. from Eq.s (6) and (11). Additional technical details can be found in Iben & Renzini (1984).

Having already mentioned the future observations which may help in deriving more accurate ages, one has to do the same on the theoretical side. The understanding of the period shift effect in terms of evolutionary models is the first challenge. Preliminarly, one would like to establish whether or not an increased [CNONe/Fe] ratio is the correct explanation for this phenomenon. If so, also isochrones with various values of [CNONe/Fe] will have to be computed. If not, it is unlikely that other changes in the input composition parameters and/or in the input physics may produce the desired effect, and the adequacy of the canonical assumptions will inevitably be called into question.

### 3.4 Two puzzling gaps

The C-M diagram of a few globular clusters exhibits one or two gaps in the distribution of stars on the main branches. Typical is the case of the C-M diagram of NGC 6752 (Cannon 1981), which shows a well defined gap at the base of the giant branch (hereafter SGB gap), and another wide gap on the blue HB (hereafter HB gap). Other examples of clusters with a SGB gap are ω Cen (Da Costa & Villumsen 1981) and NGC 288 (Buonanno et al 1983b). Canonical theoretical models do not predict the existence of a SGB gap, i.e. of an acceleration of the evolutionary rate when stars meet the red giant branch. The luminosity of this gap corresponds to an

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evolutionary stage at which the mass of the helium core (or, equivalently, the mass location of the hydrogen burning shell) has reached $\sim$0.2 $M_\odot$. The presence of the gap may perhaps indicate that the composition profile around $M_r$ = 0.2 $M_\odot$ is not the one predicted by the models, and then that some sort of mixing may have occurred in the vicinity of this region, during the previous evolutionary stages. I understand that Pierre Demarque has recently investigated this problem in some detail, so I hope that he will briefly mention his results during the discussion.

The most straightforward interpretation of HB gaps is in terms of a bimodal distribution for the mass of HB stars in a cluster. The physical origin of the bimodality, however, remains obscure (Norris 1981). Renzini (1983) has noticed that NGC 6752 and the other clusters with a HB gap (NGC 2808, Harris 1974; NGC 1851, Stetson 1981; and M15, Buonanno et al 1983a) share the common property of having unusually high central densities, and suggested that *tidal collisions* in such dense cluster cores could be responsible for the bimodal distribution of HB masses. However, Buonanno et al (1983b) have recently found that the very low density cluster NGC 288 also has a well defined HB gap. Therefore, either tidal collisions have nothing to do with HB gaps, or there are also other processes able to produce such gaps.

In any case, whatever the origin of the HB gaps, it is worth recalling that very hot HB models have such a small envelope mass ($\sim$ a few $10^{-2}$ $M_\odot$, cf Sweigart & Gross 1976), that even a modest mass loss rate during the core helium burning phase (of the order of a few $10^{-10}$ $M_\odot$/yr) is able to reverse the direction of their evolution (from redward to blueward). The possibility of such an effect should be taken into account when dealing with very hot HB stars. Indeed, even if mass loss can not by itself produce a blue HB gap, it can prevent such a gap from being filled up by evolutionary effects.

In conclusion, the origin of the two gaps still remains rather misterious, and this somewhat veils our confidence on current determinations of the age and helium abundance of globular cluster stars.

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DISCUSSION

Cox: 1) Can you get even more effect in the core mass sensitivity with Z by further changing neutrino losses?
2) Are the higher CNO abundances relative to iron consistent with recent higher O abundances that we have heard about?

Renzini: Yes. But the question is which is the correct rate of neutrino losses. A careful reexamination of the plasma process including the neutral current interaction is certainly worthwhile.
2) Yes, it is.

Rood: Did you change the core mass when you change [CNO/Fe]?

Renzini: Yes, we tried to in an approximate way.

Rood: When I made ZAHB models with different [CNO/Fe] and $M_{core}$ varying as with the standard theory I find that d log(P')/d (CNO) is almost zero.

Renzini: I should have a careful look to what you have done before drawing any conclusion. My impression is that a period-shift may appear only if [CNO/Fe] is positive and constant, but let me postpone my answer.

Itoh: We recently recalculated the neutrino-pair bremsstrahlung process using the Weinberg-Salam theory of weak interaction. The paper will soon appear in the Astrophysical Journal.

Renzini: This is very interesting, but the process dominating neutrino losses in population II red giants is the plasma process.

Demarque: 1) Although I see the attractiveness of your treatment of the ages of the globular clusters, I am concerned about the assumption that the magnitude difference between the horizontal branch and turn-off stars at the same colour is constant. It seems unclear at this point that the observations support this statement. In addition, this assumption, together with your other arguments, forces all globular clusters to have the same age. How then are we going to find out whether all globular clusters have the same age?
2) In your presentation, you alluded to our unpublished work on the sub-giant gap observed in globular cluster C-M diagrams. T. Armandroff and I

have in the last few months investigated the evolution of stars with a mixed shell in the interior (an adhoc assumption). We find that only if the mixing occurs in the subgiant phase, and only for certain thicknesses of the mixed shell, do we get a small gap approximately in the right place of the luminosity function. It is unclear at this point what relevance to the observed gap our calculations have.

Renzini: The constancy of the magnitude difference $\Delta M_{TO}^{RR}$ between the HB and the turn-off is not an assumption, it is an observed fact for all clusters with photometrically accurate C-M diagrams, including NGC 288 (Buonanno et al, Ap.J. in press) for which there were rumors for a peculiarity in this respect. This fact, together with the Sandage period-shift effect (another observed fact) ensures the constancy of the age for the studied clusters, within the quoted uncertainty. Certainly, if some day clusters will be found exhibiting substantially different values of $\Delta M_{TO}^{RR}$, then the influence will be that they have different ages.

Hesser: Concerning the gap in the blue HB in NGC 6752 and other clusters, Russell Cannon mentioned the hypothesis put forth by Norris and his collaborator that bimodal CN strengths might be correlated with the presence of a BHB gap. Unfortunately they then tested the hypothesis by observing NGC 2808 giants, which did not show the expected bimodal behavior! Thus, another observational clue about the gap apparently must be discarded.

Frogel: Is your age spread for globulars still consistent with the explanation for the "2nd parameter" effect in the distant halo objects as being due to these distant clusters having formed later than in closer clusters?

Renzini: As I said, the current uncertainty in the age determination of individual clusters is of several billion years. It is also known that the HB morphology is expected to change rather dramatically when the age is changed by just 1 or 2 billion years. Therefore, if a second parameter is required for distant clusters, the age remains a reasonable candidate.