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Numerical procedures for Volterra integral equations

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This thesis investigates new finite difference methods for the numerical solution of Volterra integral equations.

After a brief discussion of the relevant literature in Chapter 1, implicit Runge-Kutta methods, based on interpolatory quadrature formulae, are derived in Chapter 2 for Volterra integral equations of the second kind,

$$y(t) = g(t) + \int_0^t K\big(t, s, y(s)\big)ds \ , \quad t \geq 0$$

with continuous kernels $K(t, s, y)$. Convergence of the schemes is examined, and the order is found to be equal to the degree of precision of the related quadrature formula plus one. In addition, the methods are shown to be numerically stable. For certain choices of quadrature formulae they are also A-stable in the sense of Dahlquist.

In Chapter 3, the implicit Runge-Kutta methods developed in Chapter 2 are applied to Volterra integral equations of the first kind,

$$g(t) = \int_0^t k(t, s)y(s)ds \ , \quad t \geq 0 \ ,$$

where $k(t, s)$ satisfies certain smoothness conditions and $k(t, t) \neq 0$. For the schemes obtained, simple necessary and sufficient conditions for

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convergence and numerical stability are derived. From these conditions, schemes which are convergent of arbitrarily high order and numerically stable can be constructed.

Finite difference schemes for the generalized Abel equation,

$$g(t) = \int_0^t \frac{k(t,s)}{(t-s)^\alpha} y(s)ds \ , \quad t \geq 0 \ , \quad 0 < \alpha < 1 \ ,$$

where $k(t, s)$ satisfies appropriate smoothness conditions and $k(t, t) \neq 0$, are investigated in Chapter 4. Using product integration techniques, the midpoint, Euler, and trapezoidal methods for Volterra integral equations of the first kind with continuous kernels and the schemes developed in Chapter 3 are extended to this equation. Convergence results for the product integration analogues of the midpoint, Euler, and trapezoidal methods are derived.

Several of the results of this thesis have been established in collaboration with R.S. Anderssen and F.R. de Hoog . The publication details are listed in the References.

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