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Optical variability of quasars: a damped random walk

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Abstract. A damped random walk is a stochastic process, defined by an exponential covariance matrix that behaves as a random walk for short time scales and asymptotically achieves a finite variability amplitude at long time scales. Over the last few years, it has been demonstrated, mostly but not exclusively using SDSS data, that a damped random walk model provides a satisfactory statistical description of observed quasar variability in the optical wavelength range, for rest-frame timescales from 5 days to 2000 days. The best-fit characteristic timescale and asymptotic variability amplitude scale with the luminosity, black hole mass, and rest wavelength, and appear independent of redshift. In addition to providing insights into the physics of quasar variability, the best-fit model parameters can be used to efficiently separate quasars from stars in imaging surveys with adequate long-term multi-epoch data, such as expected from LSST.

Keywords. surveys, galaxies: active, quasars: general, stars: variables, stars: statistics

1. Introduction

Quasars are variable sources with optical amplitudes of several tenths of a magnitude for time scales longer than a few months (e.g., Hawkins & Veron 1995; Trevese *et al.* 2001; Ivezić *et al.* 2004; Vanden Berk *et al.* 2004). Sesar *et al.* (2007) and Butler & Bloom (2011) showed using SDSS Stripe 82 data (a $\sim$300 deg^2 equatorial region imaged about 60 times) that practically all quasars spectroscopically confirmed by SDSS are photometrically variable.

Quantitative statistical description of quasar variability is important both for understanding the physics of the driving mechanism(s), and for selecting quasars in imaging surveys. Here we describe recent progress in the analysis of quasar variability which demonstrated that a stochastic process called damped random walk (DRW) provides a satisfactory statistical description of quasar variability in the optical wavelength range.

2. Quantitative analysis of quasar variability

Two main methods have been utilized over the last few decades to quantitatively describe stochastic quasar variability: a variability structure function analysis and direct modeling of light curves.

2.1. *Structure function approach*

The structure function as a function of time lag Δt, SF(Δt), is equal to the standard deviation of the distribution of the magnitude difference $m(t_2)-m(t_1)$ evaluated at many different times t_1 and t_2, such that time lag $\Delta t = t_2 - t_1$ (and divided by $\sqrt{2}$ because of differencing; we warn the reader that a number of slightly varying definitions have been used in recent quasar studies). This operational definition has been applied to both light curves of individual objects and as an ensemble analysis tool that is applicable even when only two photometric measurements per object are available (in this case it is typically assumed that quasars selected from narrow luminosity and redshift bins have statistically the same variability behavior).

The time dependence of the structure function was found to be consistent with the prediction based on a damped random walk model (MacLeod *et al.* 2012):

$$\mathrm{SF}(\Delta t) = \mathrm{SF}_\infty \left[1 - \exp(-\Delta t/\tau)\right]^{1/2} \tag{2.1}$$

(for illustration see Figure 2 in the contribution by Ivezić *et al.* in these Proceedings). At small time lags, $\mathrm{SF}(\Delta t) \propto \Delta t^{1/2}$, and thus a DRW is equivalent to an ordinary random walk for $\Delta t \ll \tau$ (the "damped" aspect manifests itself as $\mathrm{SF}(\Delta t) \rightarrow \mathrm{SF}_\infty$ for $\Delta t \gg \tau$).

The structure function is related to the autocorrelation function, which makes a Fourier pair with the power spectral density function (PSD). The PSD for a DRW is given by

$$\mathrm{PSD}(f) = \frac{\tau^2\, \mathrm{SF}_\infty^2}{1 + (2\pi f \tau)^2}. \tag{2.2}$$

Therefore, a DRW is a $1/f^2$ process at high frequencies, just as an ordinary random walk (when $\mathrm{SF} \propto (\Delta t)^\gamma$, then $\mathrm{PSD} \propto 1/f^{(1+2\gamma)}$). The "damped" nature is seen as a flat PSD at low frequencies ($f \ll 2\pi/\tau$). A comparison of light curves drawn from a DRW and two other stochastic processes with similar PSDs is shown in Figure 1.

2.2. *Direct modeling of light curves as a damped random walk*

Observed light curves can be used to directly constrain the DRW model parameters, τ and SF_∞ (Kelly, Bechtold & Siemiginowska 2009, hereafter KBS09; Kozłowski *et al.* 2010, MacLeod *et al.* 2010, 2011, 2012; Zu *et al.* 2012). Before summarizing the main results, we briefly review the statistical properties of a DRW.

The CAR(1) process, as it is called in statistics literature, for a time series $m(t)$ is described by a stochastic differential equation which includes a damping term that pushes $m(t)$ back to its mean (see KBS09). Hence, it is also known as a DRW in astronomical literature (another often-used name is the Ornstein–Uhlenbeck process, especially in the context of Brownian motion). In analogy with calling a random walk a "drunkard's walk," a DRW could be justifiably called a "married drunkard's walk" – who always comes home to his or her spouse instead of drifting away.

Stochastic light curves can be modeled using the covariance matrix. For a DRW, the covariance matrix is

$$S_{ij}(\Delta t_{ij}) = \sigma^2 \exp(-|\Delta t_{ij}|/\tau), \tag{2.3}$$

where $\Delta t_{ij} = t_i - t_j$, and σ and τ are model parameters; σ^2 controls the short timescale covariance ($\Delta t_{ij} \ll \tau$), which decays exponentially on a timescale given by τ. The corresponding autocorrelation function is $\mathrm{ACF}(t) = \exp(-t/\tau)$. The asymptotic value of the structure function, SF_∞, is equal to 2σ. A number of other convenient models and parametrizations for the covariance matrix are discussed in Zu *et al.* (2012).

2.3. *Tests of a damped random walk model*

Both a structure function analysis and the direct modeling of light curves demonstrate that a DRW provides a good description of the optical continuum variability of quasars. For example, the time span of SDSS data from Stripe 82 is sufficiently long to constrain τ for the majority of the ~10,000 quasars with light curves (MacLeod *et al.* 2010, 2011).

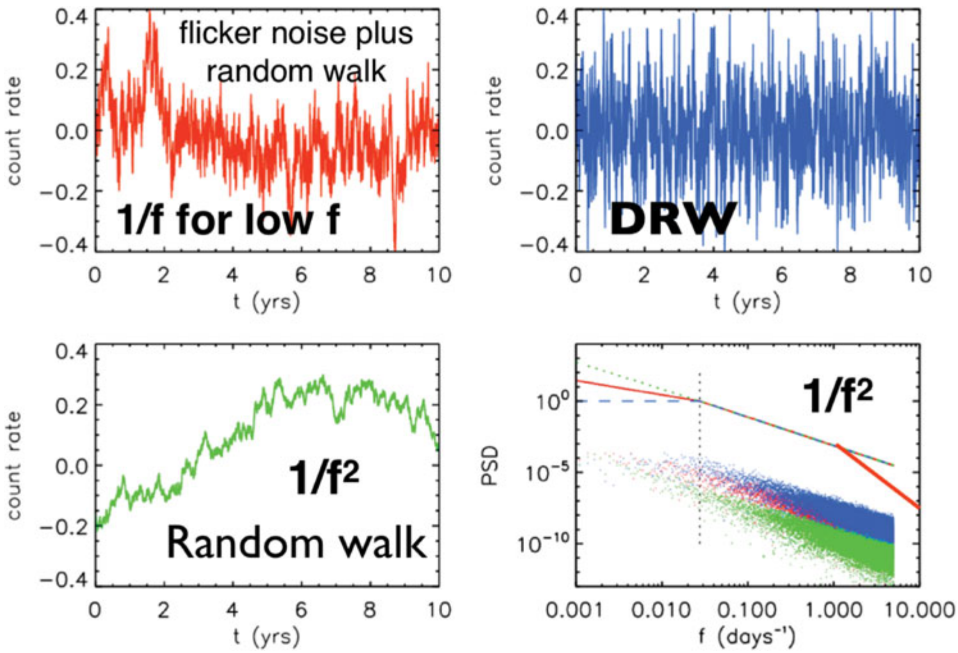

Figure 1. A comparison of simulated light curves generated using three different power spectral density functions (PSD), which are illustrated in the bottom right panel by lines (solid line: top left panel; dashed line: top right panel; dotted line: bottom left panel). In all three cases, the PSD at short time scales (large frequency f) is proportional to f^{-2} (the PSD for a random walk has the same index at *all* frequencies). The transition time scale is given by $\tau = 5.8$ days, corresponding to transition frequency $f_t = (2\pi\tau)^{-1}$ (shown by the vertical dotted line in the bottom right panel). The PSD at long time scales ($f < f_t$) follows f^{α}, with $\alpha = -1$ (top left panel, a mixture of a flicker noise and a random walk), $\alpha = -1.9$ (bottom left, almost identical to a random walk) and $\alpha = 0$ (top right, similar to a DRW). The y axis in the bottom right panel is in arbitrary units. Observed light curves of quasars are consistent with $-1 < \alpha < 0$, while $\alpha < -1$ is ruled out by SDSS Stripe 82 data. The solid line at $f > 1$ in the bottom right panel illustrates departures from the f^{-2} PSD at the shortest timescales found using Kepler data (Mushotzky *et al.* 2011). Adapted from MacLeod *et al.* (2010).

The best-fit values of τ and SF_∞ are correlated with physical parameters, as discussed in the next section.

MacLeod *et al.* (2010) have concluded that the observed light curves of quasars are consistent with PSD$\propto f^{\alpha}$ at long timescales, with $-1 < \alpha < 0$, while $\alpha < -1$ is ruled out. Furthermore, Zu *et al.* (2012) have analyzed OGLE light curves using a number of stochastic processes with covariance matrices similar to that for a DRW. They concluded that the DRW model is consistent with data on the probed time scales (from a month to a few years). Some deviations from the DRW model were detected at short timescales (a month or less) by Mushotzky *et al.* (2011) using high-precision Kepler data. They found that the measured PSD at high frequencies (from 10^{-6} Hz up to 10^{-5} Hz) is steeper than the expected f^{-2} behavior.

The distribution of magnitude differences drawn from a DRW light curve should be Gaussian. The number of points per observed light curve is typically too small to test this expectation using individual objects. When using an ensemble analysis, the observed distribution is puzzlingly closer to an exponential (Laplace) distribution than to a Gaussian distribution (Ivezić *et al.* 2004; MacLeod *et al.* 2008). Nevertheless, MacLeod *et al.* (2012) showed that the exponential distributions seen in the statistics of ensembles of quasars naturally result from averaging over quasars that are individually well described by a Gaussian DRW process.

3. Insights into the physics of quasar variability

The best-fit values of τ and SF_∞, determined using a DRW model and SDSS Stripe 82 light curves, are correlated with physical parameters, such as the luminosity, black hole mass, and rest-frame wavelength (MacLeod *et al.* 2010, 2012). Their analysis shows SF_∞ to increase with decreasing luminosity and rest-frame wavelength (as was observed previously), and without a correlation with redshift. They found a correlation between SF_∞ and black hole mass with a power-law index of 0.18 ± 0.03, independent of the anti-correlation with luminosity. They also found that τ increases with increasing wavelength with a power-law index of 0.17, remains nearly constant with redshift and luminosity, and increases with increasing black hole mass with a power-law index of 0.21 ± 0.07.

The amplitude of variability is anti-correlated with the Eddington ratio, which suggests a scenario where the optical fluctuations are tied to variations in the accretion rate, possibly in an inhomogeneous accretion disk (Dexter & Agol 2011). However, an additional dependence on luminosity and/or black hole mass was found that cannot be explained by the trend with Eddington ratio. Recent studies show evidence for enhanced color variability compared to what is expected if the mean accretion rate is solely driving the variability (Schmidt *et al.* 2012), which is consistent with a scenario involving hot spots in the disk.

4. Conclusions

The last decade has seen enormous progress in both data availability and the modeling of stochastic quasar variability. The damped random walk model provides a satisfactory statistical description for practically all the data available at this time. This progress is likely to continue thanks to new post-SDSS massive sky surveys. For example, the Large Synoptic Survey Telescope (LSST; for a brief overview see Ivezić *et al.* 2008) will extend the light curve baseline for ~10,000 quasars from SDSS Stripe 82 to over 30 years, and will obtain an additional ~800 high-precision (~0.01 mag) photometric measurements. In particular, these data will enable a definitive robust measurement of the low-frequency behavior of the PSD for quasar variability.

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