Section I

Observing Techniques and Interpretation

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COMETARY ORBITAL DYNAMICS AND ASTROMETRY

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ABSTRACT. Comets are the only large solar system bodies where nongravitational forces directly affect their dynamic motions. Their approach to within a few AU of the Sun initiates the vaporization of nucleus ices, and the resulting rocket-like effects either add to or subtract from the comet's orbital energy; the sign of the energy change depends upon the comet's rotation direction and its spin pole orientation. The cometary outgassing phenomena have generally been modeled by assuming a rapidly rotating nucleus of water ice that outgasses symmetrically with respect to perihelion. Although this nongravitational acceleration model has been quite successful in providing accurate orbits and ephemerides, several comets exhibit water production rates and visual light curves that are noticeably asymmetric with respect to perihelion. New asymmetric models are being developed that attempt to represent more closely the cometary outgassing phenomena. For the same comet, derived nongravitational parameters can differ widely, depending upon which model is used to fit the astrometric data. The uncertainties in the data and in the nongravitational acceleration model prevent realistic extrapolations of these objects' motion beyond a few hundred years, particularly if close planetary encounters are involved. Accurate orbits, ephemerides and efforts to model the nongravitational effects ultimately depend upon the quality of the astrometric data. Using a combination of long-focus telescopes, charge coupled device (CCD) detectors, microdensitometer reductions and modern star catalogs, cometary astrometric data can be generated that are accurate to the sub arcsecond level. While occultation, spacecraft, and radar observations can provide powerful astrometric data when available, it is still the ground-based optical observations that must provide the vast majority of data for cometary astrometry in the foreseeable future.

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1. **Orbital Dynamics of Comets**

1.1. STANDARD NONGRAVITATIONAL ACCELERATION MODELS

In introducing the icy conglomerate model for a cometary nucleus, Whipple (1950) recognized that comets may undergo substantial perturbations due to reactive forces or rocket-like effects acting upon the cometary nucleus itself. In an effort to represent accurately the motions of many short periodic comets, Marsden (1968,1969) began to model the nongravitational accelerations with empirical terms in the comet's equations of motion. These expressions have become known as style I nongravitational parameters. Style II parameters were added when Marsden et al. (1973) introduced what has become the standard, or symmetric, nongravitational acceleration model for cometary motions: a rapidly rotating cometary nucleus is assumed to undergo vaporization from water snow that acts symmetrically with respect to perihelion. That is, at the same heliocentric distance before and after perihelion, the cometary nucleus experiences the same nongravitational acceleration. The cometary equations of motion are written:

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\mu\,\mathbf{r}}{r^3} + \frac{\partial R}{\partial \mathbf{r}} + A_1\, g(r)\hat{\mathbf{r}} + A_2\, g(r)\hat{\mathbf{T}} \qquad (1)$$

where $g(r) = \alpha (r/r_o)^{-m}(1 + (r/r_o)^n)^{-k}$.

The acceleration is given in astronomical units/(ephemeris day)^2, μ is the product of the gravitational constant and the solar mass, and R is the planetary disturbing function. The scale distance r_o is the heliocentric distance where re-radiation of solar energy begins to dominate the use of this energy for vaporizing the comet's ices. For water ice, r_o = 2.808 AU and the normalizing constant α = 0.111262. The exponents m, n, and k equal 2.15, 5.093 and 4.6142, respectively. The nongravitational acceleration is represented by a radial term, $A_1\ g(r)$, and a transverse term, $A_2\ g(r)$, in the equations of motion. The radial unit vector ($\hat{\mathbf{r}}$) is defined outward along the Sun-comet line, while the transverse unit vector ($\hat{\mathbf{T}}$) is directed normal to $\hat{\mathbf{r}}$, in the orbit plane, and in the direction of the comet's motion. An acceleration component normal to the orbit plane is certainly present for most active comets, but its periodic nature makes a meaningful solution for it difficult in these computations because we are solving for an average nongravitational acceleration effect over three or more apparitions. If the comet's nucleus were not rotating, the outgassing would always be toward the Sun and the resulting nongravitational acceleration would act only in the anti-solar direction. However, the rotation of the nucleus, coupled with a thermal lag angle between the nucleus subsolar point and the point on the nucleus where there is maximum outgassing, introduces a transverse acceleration component in either the direction of the comet's motion or contrary to it - depending upon the nucleus rotation direction.

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Equation 2 represents the time derivative of the comet's orbital semi-major axis (a) as a result of radial and transverse perturbing accelerations (R_p, T_p).

$$\frac{da}{dt} = \frac{2}{n(1 - e^2)^{1/2}} \left[(e \sin \nu) R_p + \frac{p}{r} T_p\right] \tag{2}$$

In this equation, n, e, ν, and r denote, respectively, the orbital mean motion, eccentricity, true anomaly, and the comet's heliocentric distance, while p is the orbital semi-latus rectum, $a(1-e^2)$. The orbital true anomaly is illustrated in Figure 1.

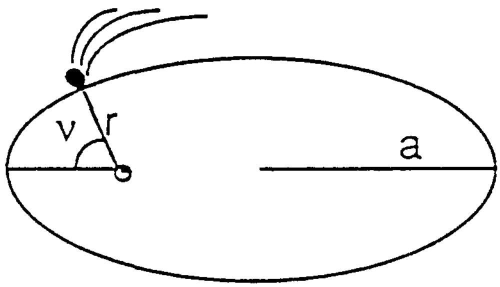

Figure 1. The cometary ellipse showing the true anomaly (ν).

Because of the thermal lag angle, a comet in direct rotation will have a positive, transverse nongravitational acceleration component, and from equation 2, it is apparent that the comet's orbital semi-major axis will increase with time (its orbital energy will increase). Likewise, a comet in retrograde rotation will be acted upon by a negative T_p and its semi-major axis will decrease with time. Because the nongravitational acceleration is assumed to act symmetrically with respect to perihelion, the time-averaged effect of the periodic radial acceleration is far less than that of the transverse component, which acts in a secular fashion.

When introducing the standard model, Marsden et al. (1973) included possible time dependences in the transverse parameter (A_2). Currently, however, the standard nongravitational acceleration model is most often used solving only for the radial and transverse parameters (A_1 and A_2) over data arcs short enough so that neglected time dependences do not cause systematic trends in the residuals. However, determinate solutions for the nongravitational parameters require data from at least three apparitions. By comparing the nongravitational parameters, determined from several of these short arc solutions, one can determine their change with time. A determination of a nongravitational parameter normal to the orbit plane (A_3) does not generally improve an orbit solution significantly, although Sitarski (1984) showed there was some improvement for the motion of comet Kopff. Subsequently, Rickman et al. (1987) were able to link successfully five apparitions of comet Kopff, a comet well-known for time-dependent nongravitational accelerations, by solving for A_1, A_2, A_3, and allowing the latter two to vary linearly with time. Although the inclusion of additional free parameters in the solution set (such as A_3 and time dependences) will likely result in lower overall mean residuals, great care must be taken not to place undue physical meaning upon the resultant values and their formal errors. In addition, the prediction capability of the orbit may not be improved over shorter arc solutions using fewer variables in the solution set.

Whether or not the standard, symmetric model accurately represents the cometary outgassing phenomena that is responsible for the nongravitational effects, it has succeeded rather well in improving the ephemeris prediction accuracies for short-period comets. Recovery observations at the beginning of each cometary apparition indicate the correction required to the predicted perihelion time; when the standard nongravitational model is used for a comet with an extensive observation history, this correction is rarely more than a few tenths of a day. Orbital solutions that include determinations for the parameters A_1 and A_2 have been made for most well-observed, short-period comets, and Marsden (1985, 1986) has cataloged these efforts.

For at least eight long-period comets that have been observed for only one apparition, satisfactory fits to the astrometric data require a solution for nongravitational parameters. However, an improved fit to the data does not necessarily imply an improved ability to represent the comet's future motion, and without the opportunity to observe these comets on subsequent returns, it is difficult to determine whether or not their nongravitational parameters are physically meaningful or even helpful in representing the comet's long-term motion. In this regard, mention should also be made of attempts to determine nongravitational effects for objects that are referred to as asteroids but may well be nearly inactive comets (Ziolkowski, 1984). Here the problem is whether the improved data fits with nongravitational effects are true improvements or simply the expected reduction in the mean residual due to the addition of free parameters in the solution set.

1.2. ALTERNATE NONGRAVITATIONAL ACCELERATION MODELS

Long before, and even after, the introduction of the standard nongravitational acceleration model, there have been alternative attempts to model these anomalous effects. Encke (1820) postulated a resisting medium to explain the motion of the comet that bears his name, and more recently Brady and Carpenter (1971) used the device of decreasing the radial solar acceleration linearly with time to represent the motion of comet Halley. Sitarski (1981) used the traditional secular variation in mean motion, but incorporated this device directly in the comet's equations of motion expressed in rectangular coordinates. While these devices could satisfactorily represent the existing observational data and even provide satisfactory predictions of cometary motion, there was no realistic physical model upon which they were based.

Rickman and Froeschlé (1982, 1986) and Froeschlé and Rickman (1986) developed a thermal model of the cometary nucleus and used it to investigate how the cometary nongravitational accelerations are affected by changes in their model parameters, and in particular, the comet's thermal inertia. They find that the radial, transverse, and normal components at a given orbital position are sensitive to the comet's assumed thermal inertia. This is particularly true for the transverse component that is primarily responsible for introducing secular effects in the standard, nongravitational model. Their heat flow models do not predict any general acceleration law of the same type used by the standard model. That is, each of the three nongravitational acceleration components in their thermal models has a different variation with heliocentric distance, so that the expression $g(r)$ used in equation 1 above is not generally compatible with their results.

For comet Halley, Landgraf (1986b) attempted to use Rickman and Froeschlé's thermal models to improve the data fit over the period from 1607 through 1985. Solutions were made over a range of a factor of eight in thermal inertia and a factor of five in the comet's rotation period, but the resultant mean residual was identical to that obtained using the standard model. Reverting back to the standard model, Landgraf also made solutions for A_1, A_2, and A_3 values for three different segments of comet Halley's orbit, thereby attempting to determine their change with orbital position. In particular, the variation in A_3 was used to determine that the location of the comet's spin pole was nearly in the orbital plane, a result differing markedly from that determined using spacecraft camera data (Sagdeev et al., 1989).

Yeomans (1984) attempted to model comet Halley's nongravitational effects by scaling them according to the asymmetric 1910 light curve, and varying the three cometary nucleus orientation angles employed by Sekanina (1981). While the minimum root mean square (RMS) observation residual was reached with a spin pole obliquity of 30 degrees to the orbit plane normal, within 10 degrees of the value derived from the spacecraft images (Sagdeev et al. 1989), the symmetric acceleration model still provided a better overall fit to the data. Hechler et al. (1986) pointed out that improved orbit results could be obtained for comet Halley with a radial nongravitational acceleration that was scaled with respect to the asymmetric 1910 light curve. However, when Hechler et al. attempted to improve the orbital solutions, using thermal models more sophisticated than the standard model, they were largely unsuccessful.

Yeomans and Chodas (1989) modified the standard nongravitational acceleration model to allow the water vaporization curve to peak a certain number of days (DT) either before or after perihelion. When compared with the standard model, the new, asymmetric model often improves the data fit. The best fit to the astrometric data generally occurs for a value of DT corresponding to the offset in the comet's visual light curve. This asymmetric model, which more accurately mimics the comet's outgassing history, often yields values for the radial and transverse nongravitational parameters that are completely different from corresponding values derived using the standard symmetric model; thus these latter quantities are extremely uncertain and works that have employed them to infer physical properties of comets will have to be re-examined. Rotation and precession characteristics of cometary nuclei can differ markedly, depending upon the model used. For example, the asymmetric model suggests only weakly that the rotation direction of comet Halley is direct and there is no longer any evidence of Yeomans' (1974) suggestion that the rotation axis of comet Kopff passed through its orbit plane in the early twentieth century. Because the asymmetric model relies primarily upon the radial, rather than the transverse, component to create secular changes in a comet's nongravitational motion, the results with the asymmetric model generally will yield lower thermal lag angles.

1.3. EXTRAPOLATIONS OF ORBITAL SOLUTIONS

The accuracy of long-term numerical integrations of cometary motions is sensitive to uncertainties in the initial conditions, the nongravitational acceleration model, and the masses and positions of the perturbing bodies. Close planetary encounters can dramatically increase a comet's orbital uncertainty so that the continuation of an integration beyond such an encounter is often of only statistical interest, since any knowledge of the comet's true path is lost. With good initial conditions, and in the absence of close planetary encounters, the realistic extrapolation of a particular comet's motion forward or backward in time can be followed for a few hundred years, but certainly not for a few thousand years.

Beginning with her publications in 1967, Kazimirchak-Polonskaya numerically integrated several real and hypothetical cometary orbits over the interval 1660 - 2060 to investigate the transformation of cometary orbits due to perturbations by the major planets. One major result of her work was the demonstration that perturbations by an outer planet can modify a comet's orbit so as to bring it under the perturbative influence of the next planet closer to the Sun. Presumably the process can continue until the comet is captured into the Jupiter family of periodic comets (see Kazimirchak-Polonskaya, 1985 and references therein).

Carusi et al. (1985) integrated the motion of the 132 short-period comets known at that time over the interval 1585 - 2406. Carusi et al. were able to find several orbits that are chaotic and several that librate about planetary resonances. A few temporary captures into planetary satellite orbits were also identified (for example, P/Schwassmann-Wachmann 2, P/Oterma, and P/Gehrels 3). The orbits of comets Neujmin 3 and Van Biesbroeck, shown in Table 1, were found to be nearly identical prior to their 1850 Jupiter close approach, and it seems likely that these two comets were split off from the same parent object sometime prior to 1850. In this regard, mention should also be made of comets 1988e Levy and 1988g Shoemaker-Holt, whose very similar orbits suggest that these two comets may well have originated with the same parent object as a result of a split many years ago (Marsden, 1988). Apart from the Kreutz sungrazers, this latter pair of comets is the only likely example of long-period comets with a generic connection.

TABLE 1. Comets with possible common parents. Angular elements are referred to the ecliptic plane and the 1950.0 equinox.

P/Neujmin 3 (after Carusi et al., 1985)

Epoch	q(AU)	e	ω	Ω	i
1851	2.1838	0.5742	132.12	162.79	3.17
1848	2.7495	0.5506	290.72	348.54	4.72

P/Van Biesbroeck (after Carusi et al., 1985)

Epoch	q(AU)	e	ω	Ω	i
1851	2.4531	0.5554	123.57	163.89	7.01
1848	2.7578	0.5516	289.77	349.14	4.12

Levy (1988e) (after B.G. Marsden, M.P.C. 13452)

Epoch	q(AU)	e	ω	Ω	i
1987	1.1741	0.9978	326.52	288.06	62.80

Shoemaker-Holt (1988g) (after B.G. Marsden, M.P.C. 13452)

Epoch	q(AU)	e	ω	Ω	i
1987	1.1745	0.9978	326.52	288.07	62.81

The catalogue of short-period comets by Belyaev et al. (1986) presents the results of numerical integrations for all short-period comets known through the end of 1983. For comets with more than one apparition, orbital data are given over the interval 1800 - 2000; for the six periodic comets discovered prior to 1800, the interval of investigation was pushed back to 1750. The host of auxiliary information provided for each comet, together with the comprehensive appendices, makes this a particularly useful reference work, and the inclusion of the standard nongravitational effects assures accuracy over their integration intervals.

By constraining the motion of comet Halley with ancient Chinese observations back to A.D. 141, Yeomans and Kiang (1981) were able to integrate successfully the motion of this comet back to 1404 B.C. Subsequently, Babylonian observations of the comet in 87 B.C. and 164 B.C. were discovered and found to be consistent with the comet's computed motion (Stephenson, Yau, and Hunger, 1985). Using a slight time dependence in the comet's nongravitational effects, Landgraf (1986b) also integrated the motion of comet Halley backwards in time, making use of the ancient Chinese observations to produce orbital elements very similar to those of Yeomans and Kiang back to 466 B.C., when

Landgraf's integration was terminated. Chirikov and Vecheslavov (1989) used a simple model for the orbital evolution of comet Halley and suggested that the motion of comet Halley is chaotic. If true, then extrapolations of Halley's motion may not be possible for more than a few hundred years beyond the observation interval used to generate the initial conditions.

2. Astrometry

2.1. OBSERVING TECHNIQUES AND THE DATA REDUCTION PROCESS

Ultimately, any study of cometary motions depends rather strongly upon the quality of the astrometric data employed. Traditionally, cometary astrometry has been used to generate orbits and ephemerides to aid in the recovery of a particular comet and to provide ephemerides for astronomers who wished to make physical observations of the comet. More recently, however, cometary astrometric data have been used directly to help characterize the cometary outgassing phenomena. Within the body of astrometric data for solar system objects, cometary observations are unique in that they must be used to help define the nongravitational acceleration model itself. The development of increasingly more sophisticated models for cometary nongravitational effects cannot succeed without a parallel increase in astrometric accuracy.

General guidelines for astrometric techniques with various types of instruments have been well-documented by Russell (1984), Everhart (1984), Mrkos (1984), Belton (1984), McCrosky (1984), West (1984), and Gilmore and Kilmartin (1984). Gibson (1984), Harrington (1984), and Stock (1984) have given detailed discussions of plate reduction techniques. The following general recommendations for taking and reducing cometary astrometric observations are based upon these works.

1. Use the shortest useful exposure that is consistent with a good signal-to-noise ratio.
2. A red filter is preferred.
3. Good tracking is required (preferably on the comet rather than on the stars).
4. Measure UTC times of exposure start and stop times to one second.
5. Observations should include the observer's name, the telescope used and its location, the image scale, sky/weather conditions, a subjective assessment of image quality, problems encountered, and estimates of the nuclear/total magnitudes (but only if reliable).
6. Use at least twice as many reference stars as there are terms in each coordinate (i.e., 6 stars for linear reductions with astrographs and long-focus reflectors, and 12 to 15 stars for Schmidt telescope observations). Avoid reductions using only three stars.

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7. Measure the point of peak brightness and try to expose such that the image of the comet is one magnitude above the threshold.
8. Reject dubious images.
9. Use a least-squares plate constants method, rather than dependences.
10. Give topocentric (not geocentric) B1950 positions, generally with the UTC time given to 0.00001 day, R.A. to 0.01 seconds of time, and Dec. to 0.1 second of arc.
11. Do not make corrections for elliptic aberration.
12. If measurements are being made by the bisection method, take both direct and reverse readings of the micrometer, with the target both first and last; for the automatic scanning method, it is sufficient to record the readings in one direction.

Within a few years, the comet and asteroid community may well transfer from the B1950 to the J2000 coordinate system, but currently the necessary reference star catalogs are either not available or not easily accessible to most observers. At least for the northern hemisphere, this problem may soon be eliminated, since Bastian and Röser (1988) have recently completed their Positions and Proper Motions (PPM) star catalog containing a total of 181,731 stars included on the FK5 - J2000 system. When northern and southern hemisphere versions of this catalog are widely available, the star positions accuracies will improve to a level (approximately 0.3") that many current catalogs cannot provide.

The use of CCDs and microdensitometer measurements of astrometric images is becoming more widespread. CCD-equipped telescopes, when used in combination with reductions that are carried out with microdensitometer scanning and image processing, can routinely provide astrometric accuracy below one arcsecond for all but the most active comets (for example, see West, 1984). However, for active comets, the center of light may not correspond to the comet's center of mass.

2.2. POSSIBLE OBSERVING BIASES

Because of rather systematic trends in comet Halley's orbit residuals during March to April 1986, it was necessary to model an observation bias to obtain solutions that fit the observations to the level of the data noise itself. Yeomans and Chodas (1989) followed Yeomans et al. (1983) and assumed that the comet's center of mass was offset a distance (S) radially away from the Sun from the observed center of light (see Figure 2). This measurement bias, S, varies as the inverse square of the heliocentric distance, and the expression was normalized to a heliocentric distance of one AU (i.e., at $r = 1$ AU, $s = s_o$).

The offset of comet Halley's center of light from its center of mass at one AU (s_o) was included in the solution set, and the offset was assumed to be operative during each apparition in the solution. Using either of two different data intervals (1835 to 1989 or 1759 to 1989), the same value of s_o = 880 km resulted. Landgraf (1986a, 1986b) modeled this light shift by assuming its magnitude is directly proportional to the geocentric distance and inversely proportional to the square root of the reduced magnitude. Using this model,

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he estimated the light shift at one AU from the Sun to be some 3700 km. Landgraf (1986b) also attempted to avoid the sunward observation bias altogether by transforming some of his observations from right ascension and declination to position angle and apparent angular distance to the Sun, thus rendering his observation set insensitive to sunward observation biases. However, he found the mean errors increased when this technique was employed in orbital solutions.

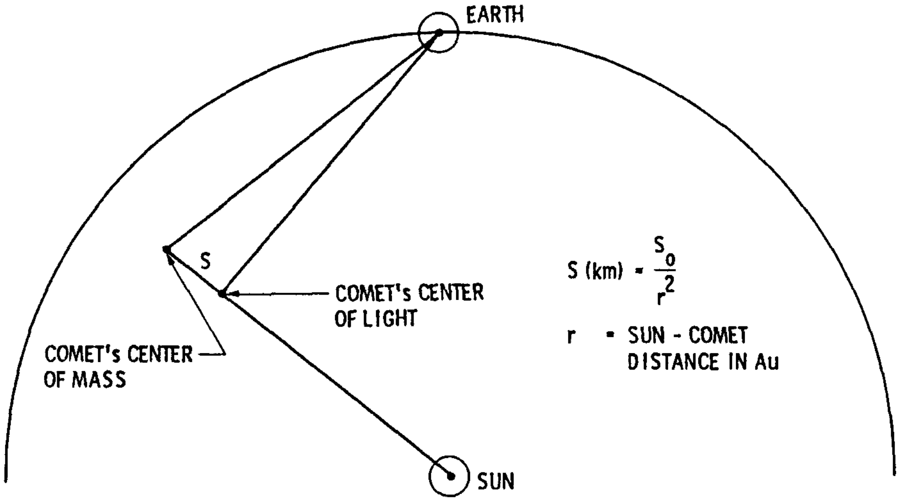

Figure 2. An observational bias introduced by an offset of the comet's center of mass from its photometric center of light.

It is not entirely clear whether the post-perihelion observation bias observed for comet Halley is due to an actual offset between the comet's center of mass and its center of light or whether it is due to instrumental effects. Röser et al. (1986) pointed out that the position of a comet's center of light will depend upon the focal length of the telescope used, the exposure time, and the type of measurement (either by eye or by scanning machine). It is worth noting that the astrometric positions resulting from the longest focal length instrument employed within the Astrometry Network of the International Halley Watch did not show an obvious post-perihelion residual bias (the 0.61-m, f/14 Cassegrain used by A. C. Gilmore and P. Kilmartin at Mt. John Observatory). After analyzing the 1984-86 orbit residuals, Diaz-Bobillo and Zadunaisky (1988) pointed out a possible observation bias component that was normal to the orbit plane, suggesting that a radial-only model may be inadequate. Clearly, there is an incomplete understanding of the observation bias observed in comet Halley's orbit residuals.

2.3. NEW DATA TYPES

The steady improvement in star catalog accuracy, together with sophisticated observing and reduction procedures, allows current cometary astrometric observations to achieve arcsecond accuracy. Special star catalogs were generated for the path of comet Halley (Klemola et al., 1984; Holdenreid and Crull, 1986), enabling sub arcsecond accuracy for some observers who took advantage of these catalogs. Space telescope and/or occultation observations will allow improved angular accuracy for the relative star - comet offsets, but the inertial astrometric position will still depend upon the accuracy of the star positions. Even if highly accurate, inertial, angular observations are forthcoming, two or more of them will be required to improve the knowledge of a particular comet's mean motion - a requirement for improving its long-term ephemeris.

The success of the Pathfinder Project has been well-documented (Münch et al., 1986) and it is of obvious interest to see if these spacecraft data can improve the long-term orbital motion of comet Halley. VEGA 1 and VEGA 2 camera pointing angle data were included in the observation data set used for Halley's orbit determination (Morley, 1986; Yeomans, 1986). Yeomans used some 232 sets of spacecraft-comet angles from VEGA 1, as well as another 236 sets from VEGA 2. As part of the Giotto navigation effort, Morley and Hechler (1984) and Morley (1986) extensively studied the Halley orbit solutions that included the Pathfinder observations. Morley (1986) and Yeomans (1986) found these solutions provided excellent "local" ephemeris predictions for the times of the spacecraft encounters in March 1986. However, these Pathfinder solutions cannot represent the astrometric data from 1835 to late 1986 without introducing systematic residual trends either in the ground-based data (rising to 30" to 50" during the close Earth approaches in 1835 and 1910) or in the Pathfinder observations themselves.

Radar data of close Earth approaching asteroids and comets have been shown to be an extremely powerful data type, especially for objects without a lengthy history of traditional optical astrometry (Yeomans et al., 1987; Ostro et al., 1989). Though cometary radar data that can be used for astrometric purposes are very limited, several near-Earth asteroids (extinct comets?) have been observed with radars from both the Arecibo and Goldstone observatories. From an ephemeris improvement standpoint, the optimal time to make Doppler astrometric observations is when the Earth-asteroid radial velocity is largest - unfortunately not at the closest approach point, where echoes are strongest. When only one Doppler observation is made, the sensitivity of the ephemeris uncertainties to the Doppler accuracy is not as impressive as when two Doppler observations are made. Future ephemeris uncertainties are very sensitive to the addition of one range observation, but there is little sensitivity to range accuracy and only a modest sensitivity to the addition of a second range observation.

Once comets or asteroids are recovered optically on their second (and subsequent) apparitions, their orbits are vastly improved and generally secure, with or without radar observations. However, for recently discovered objects, radar data can easily make the difference as to whether or not the object is recovered at its next return. Even for objects with very secure orbits, radar astrometry can still provide a several-fold improvement in the accuracy of future ephemerides. This improvement would be valuable for occultation predictions or dynamical studies, or if the asteroid should become a target for a future flyby or rendezvous space mission.

While astrometric observations of comets using radar or space-based platforms are potentially important observations, ground-based astrometric observations will continue to provide the solid foundation upon which dynamic and physical models are based. One should bear in mind that without the dedicated efforts of a small group of astrometric observers, who apply their precise art year after year with very little recognition, there would be no models, no orbits, no ephemerides, and virtually no physical observations of comets - or any other celestial object.

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