deeply than do Enderton or Leary into such things as quantifier elimination and nonstandard models of arithmetic and analysis, and he also provides some fairly sophisticated applications, including the Ax-Grothendieck theorem on polynomial maps.

Axiomatic set theory comprises two chapters and about 100 pages of text, or about one-fifth of the book. Most introductory logic textbooks don't cover this topic at all, so the presence of this material definitely enhances the versatility of this book. By and large the discussion here is clear and well written, but some choices made by the author puzzle me. Although, for example, he refers to the axiom of choice as 'infamous', he doesn't really give much reason why. Likewise, the Continuum Hypothesis receives rather short shrift, just a brief mention with one sentence pointing out that it can neither be proved nor refuted from the axioms of set theory. By contrast, perfect sets receive several pages of discussion, presumably so that the author can show how set theory can say things about the real numbers. This seems a rather large detour to me, and I can't help but feel that some of these pages would have been better spent, for example, discussing some of the interesting history of the Axiom of Choice.

The final part of the book covers completeness and decidability. Computable functions are defined in two ways, that are later shown to be equivalent: inductive generation and by means of unlimited register machines (similar to, but not the same as, Turing machines). Godel's two incompleteness theorems (for language of semirings) are proved.

In summary: this is an interesting and well-written text, but may be a bit too demanding for many undergraduates.

10.1017/mag.2024.148
Published by Cambridge University Press
on behalf of The Mathematical Association

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50 mathematical ideas you really need to know by Tony Crilly, pp 209, £9.99 (paper), ISBN 978-1-52942-804-9, Quercus Publishing plc (2023)

This lovely little book provides a concise overview of some of the most important mathematical ideas developed over the centuries. Of course, it is a personal, subjective selection by the author who undertook a mammoth task in attempting to condense into 50 short vignettes some of these developments, but he has succeeded fairly well in his mission.

It deals with 50 short, digestible snippets about ideas involving counting, number theory, geometry, algebra, relativity, game theory, chaos, fractals, graph theory, probability, coding theory, non-Euclidean geometry, linear programming, Fermat's Last Theorem, and so on. Right at the end there is a short discussion of the Riemann Hypothesis. At the end of each section there is a short historical overview, and, where possible, the author also mentions some real world applications.

The book is not aimed at the specialist but rather at the general public, teachers, parents, and school students. The over-arching intention is to create interest in the broad scope and applicability of mathematics to many aspects of modern day life. It is hoped that some high school students will become intrigued by some of the topics, and become motivated to study mathematics further.

No advanced knowledge of mathematics is required, and the reading is easy and non-technical. It is therefore highly recommended as a school library asset or for use by a mathematics teacher or parent who wants to show their learners some of the beauty and power of mathematics that goes beyond the narrow confines of the curriculum. It would also make a nice tea table display copy for casual reading by anyone who might want to gain a better appreciation of the richness and variety of modern mathematics and its cultural significance.

My only criticism is that the book could have been even more useful to novices if it had contained some extended bibliography for interested students to explore some of the ideas further. Nowadays references could even have been made to the mathematical content given in *Wikipedia*.

10.1017/mag.2024.149

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The art of mathematics – take two by Béla Bollobás, pp 334, £19.99 (paper), ISBN 978-1-108-97826-2, Cambridge University Press (2022)

When I was at school my wonderful maths teacher Ian Harris gave me a wonderful book called *Geometry for Sixth Forms* by Tuckey and Swan. In that book there is a section called 'SELECTED RIDERS' with the tempting headline 'Mostly *hard*, for those who like them *hard*.' Those italics have stayed with me ever since, and I think they apply very neatly to the book under review. The subtitle 'Tea time in Cambridge' mirrors that of the 'Take One' edition (2006) '*The Art of Mathematics – Coffee Time in Memphis*' (where the author holds a chair in combinatorics in addition to his Cambridge fellowship) and refers to the kind of problems which might be discussed among staff and students relaxing during a refreshment break. (Is this delightful practice dying out in these pressurised times?) This is certainly not a 'popular maths' book of puzzles; possibly some among the general mathematically literate public would be put off by Problem 1 which asks (the easy bit) 'Let $n \geqslant 1$ be a fixed natural number. Suppose that $0 < x_1 < x_2 <\ldots < x_N < 2n + 1$ are such that $|kx_i - x_j|$ for all natural numbers i, j and k with $1 \leqslant i < j \leqslant N$. At most how large is N?' Problem 2 on the existence of Egyptian fractions is much more down-to-earth, something of a relief to an ordinary mathematician like me.

There are 128 problems altogether, each with its own Hint and Solution; these are enlightening and often carry tangential information which is of significant interest in itself, including references to versions of the problems, anecdotes about those involved and pen sketches by Gabriella Bollobás. The extreme variety makes a general survey impossible but here are a few samples.

Geometry problems with 'adventitious angles' (which happen to allow others to be calculated without any use of trigonometry) are represented by two (one of which, for me, goes back to those wonderful SELECTED RIDERS), Both involve an isosceles triangle with extra lines in which so many angles are given that surely finding that angle should be easy—but isn't. There are problems with geometrical and combinatorial flavour, for example 'Let S be a set of $2n + 1 \geqslant 5$ points in the plane (no three on a line and no four on a circle). A circle C 'halves' S if 3 points are on C, and $n - 1$ inside and $n - 1$ outside C. Show that there are at least $n(2n + 1)/3$ halving circles.' This is done by showing that every pair of points is on a halving circle—'on the easy side for this volume' admits the author, but never mind, it is followed up by a tougher one, asking that for every n there is such a set of points with exactly n^2 halving circles. There are many linked problems, for example 'For a prime $p \geqslant 3$ the equation $p^m = r^n - 1$ has no solutions in positive integers m, r, n all > 1'.