Solar cycle variability induced by stochastic fluctuations of BMR properties and at different amounts of dynamo supercriticality

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Abstract. Understanding the irregular variation of the solar cycle is crucial due to its significant impact on global climates and the heliosphere. Since the polar magnetic field determines the amplitude of the next solar cycle, variations in the polar field can lead to fluctuations in the solar cycle. We have explored the variability of the solar cycle at different levels of dynamo supercriticality. We observe that the variability depends on the dynamo operation regime, with the near-critical regime exhibiting more variability than the supercritical regime. Furthermore, we have explored the effects of the irregular BMR properties (emergence rate, latitude, tilt, and flux) on the polar field and the solar cycle. We find that they all produce considerable variation in the solar cycle; however, the variation due to the tilt scatter is the largest.

Keywords. Bipolar Magnetic Regions, Polar Field, Solar Cycle, Solar Dynamo

1. Introduction

The irregular variation of the solar magnetic cycle is the characteristic feature of the solar magnetic field with a periodicity of about 11 years. In addition to this, short-term variations, often referred to as the "seasons of the sun," are also observed in the cycle (Rieger et al. 1984; Gurgenashvili et al. 2016; Biswas et al. 2023b). These variations manifest in phenomena such as double peaks (Gnevyshev peaks) in the solar cycle, timing of polar field reversal, duration of polar field and solar cycle, active regions, extended periods of low and high solar activity, and total solar irradiance (Karak et al. 2018; Mordvinov et al. 2020; Mordvinov et al. 2022). Therefore, understanding the variable nature of the solar magnetic cycle is essential, as it has direct and indirect impacts on human society. (Petrovay 2020).

The underlying dynamo process in the sun can explain the variability of the cycle (Karak et al. 2014a; Karak 2023). It is believed that solar dynamo is $\alpha\Omega$ type, and the magnetic field grows when the dynamo number exceeds a critical value which is defined by,

$$D = \frac{\alpha_0 \Delta\Omega R_\odot^3}{\eta_0^2},\tag{1}$$

where α_0 is the measure of the α effect, η_0 is the turbulent magnetic diffusivity, $\Delta\Omega$ is angular velocity variation, and $R_\odot$ is the solar radius. There is a huge amount of literature on the understanding of solar cycle variability based on the dynamo process (Charbonneau 2020; Karak 2023) and, which can be broadly classified as 1) nonlinear dynamics and deterministic chaos 2) stochastic forcing of the dynamo. In the

Babcock–Leighton dynamo scenario, the nonlineary primarily comes from the flux loss due to magnetic buoyancy (Biswas et al. 2022), latitude quenching (Karak 2020), and tilt quenching (Jha et al. 2020). The variability in the magnetic cycle comes from the fluctuating part of the Babcock–Leighton dynamo, i.e., from the Babcock–Leighton mechanism of polar field generation, which involves randomness. This extensive randomness arises in the Babcock–Leighton mechanism from the irregular properties of Bipolar magnetic Regions (BMRs), mainly from tilt scatter (Olemskoy and Kitchatinov 2013; Jha et al. 2020). The other sources of irregularity in the solar dynamo of observed BMR properties are time delay in BMR emergence, variation in flux, and BMR emergence latitude. The time delay of successive BMR emergence depends on the strength of the toroidal magnetic field in the convection zone (Jouve et al. 2010) and may cause solar cycle variation. Observations suggest that the mean latitude of BMR emergence on the solar surface is not uniform from cycle to cycle and depends on the strength of the cycle (Mandal et al. 2017), which can lead to solar cycle variability. The flux of BMRs is also not constant and varies irregularly throughout the cycle in the range 10^{21} to 10^{23} Mx (Sreedevi et al. 2023) and can produce variation in the solar cycle.

2. Model and Method

To see the effect of irregular properties of BMR and supercriticality on the solar cycle variation, we used 2D and 3D dynamo models. We used the Surface Flux Transport And Babcock–Leighton (STABLE) dynamo model to incorporate the observed irregular properties of BMR in the model. For a detailed description of the model, see (Karak and Miesch 2017). The STABLE dynamo model can be used as the Surface Flux Transport (SFT) model and the dynamo model. To use it as the SFT, we feed the synthetic BMR data into STABLE and see the evolution of the polar field. We generated the synthetic BMR through synthetic BMR generation code following the Jiang et al. (2018); also see (Kumar et al. 2023). The BMR produced by the STABLE model and synthetic BMR generation code follows the following properties. 1) The time delay of BMR emergence roughly follows the lognormal distribution. 2) The flux variation is also lognormally distributed. However, to see the solar cycle variation at different level of supercriticality, we used the Babcock–Leighton type flux transport dynamo model (Karak et al. 2014a; Charbonneau 2020). For a detailed understanding of the models used in the study, see Kumar et al. (2021a).

3. Results

In this section, we will demonstrate the solar cycle and polar field variability by utilizing 2D and 3D dynamo simulations. Initially, we will explain dynamo supercriticality's influence on the solar cycle's variation. Subsequently, we will elucidate the effects of irregular BMR properties on the polar field and its consequential impact on cycle variability.

3.1. *Variability in the different operation regime of the dynamo*

We shall first examine the solar cycle variability across different levels of supercriticality in the dynamo, as illustrated in Figure 1. Figure 1 (a and c) depict the variation in the solar cycle in the near-critical or weakly supercritical regime, while (b) and (d) illustrate the supercritical regime. The analysis of Figure 1 reveals that when the dynamo operates in the weakly supercritical regime, it induces large variability in the cycle and extended episodes of grand minima (Cameron and Schüssler 2017; Kumar et al. 2021a; Vashishth et al. 2023). On the other hand, when the dynamo operates near the supercritical regime, the variability in the cycle is notably reduced. The cause of this is that when the dynamo operates near the critical regime, the inherent nonlinearity of the
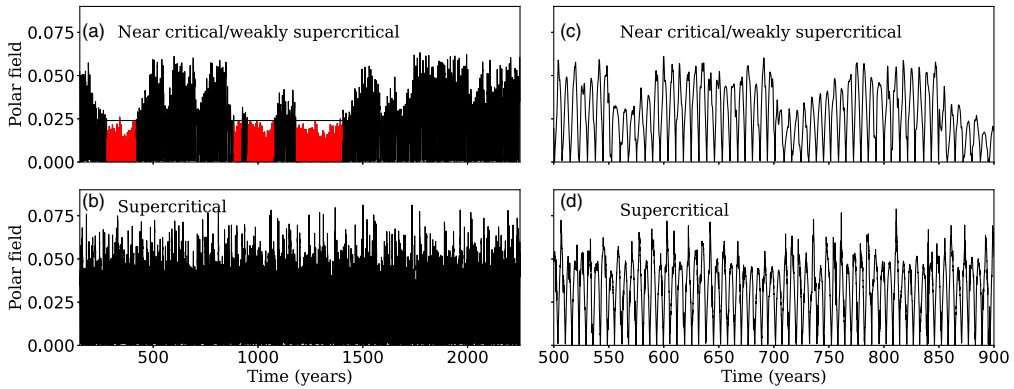
Figure 1. The plot shows the variation in the solar cycle from a dynamo simulation which operates in the weakly supercritical region (a and c) and in the supercritical region (b and d). **c** and **d** show 400 years of data from the long Run **a** and **b**. The red color shows the extended episodes of grand minima in weakly supercritical regime. The plot are produced from the model used in Kumar et al. (2021a).

dynamo weakens. Consequently, even a slight change in the dynamo number (D) results in small magnetic field growth, as the impact is linear. Thus, after some time, when D is increased due to fluctuations, the magnetic field experiences linear growth. Therefore, a large variability is observed in the magnetic field when the dynamo operates near the critical regime. Conversely, in the supercritical regime, the dynamo's nonlinearity is strong. In this scenario, significant changes in D do not translate into substantial growth in the magnetic field, as the nonlinearity suppresses such effects. Thus, in the supercritical regime, despite the large dynamo number, the variation in the magnetic field is observed less than the critical regime; also see (Karak et al. 2015; Vashishth et al. 2021; Karak 2023).

3.2. *Variation in the polar field*

Now we shall discuss the variation in the polar field and in the solar cycle due to irregular properties of BMR. Using the STABLE dynamo model as SFT, we see the variation in the polar due to different irregular properties of BMR. We see variations due to time delay in successive BMR emergence for different random realizations. Due to time delay, we see a negligible variation in the polar field; see Figure 1. The reason for the negligible variation in the polar field due to time delays is that the maximum delay in BMR emergence is much smaller than the transport time of the flux towards the pole from lower latitudes. Consequently, the variation in the polar field due to time delay is smoothed out by the time required for the flux to reach the pole. As a result, the net change in the deposited flux in the polar region is negligible due to the smoothing effect of this transport time.

Figure 3 shows the variation in the polar field due to fluctuation in BMR flux, latitude, scatter in tilt, and due to overall fluctuation, (a), (b), (c), and (d), respectively. We see that the maximum variation in the polar field is produced due to scatter in the BMR tilt. However, fluctuation in mean BMR latitude and flux variation also produces a significant amount of variability in the polar field; see Figure 3. The cause of the variation due to these parameters is the following in brief. The anticipated variation in the polar field due to latitude variation stems from the fact that when BMRs emerge at low latitudes, the cross-equatorial flux cancellation is notably more effective compared to those at
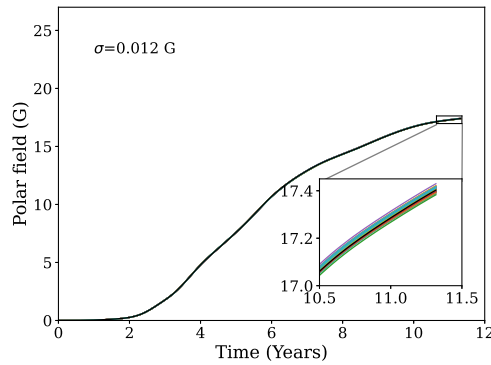
Figure 2. The plot illustrates the impact of time delays in successive BMR eruptions on the polar field. The black curve shows a reference polar field in the plot, with a zoomed-in view provided in the inset.
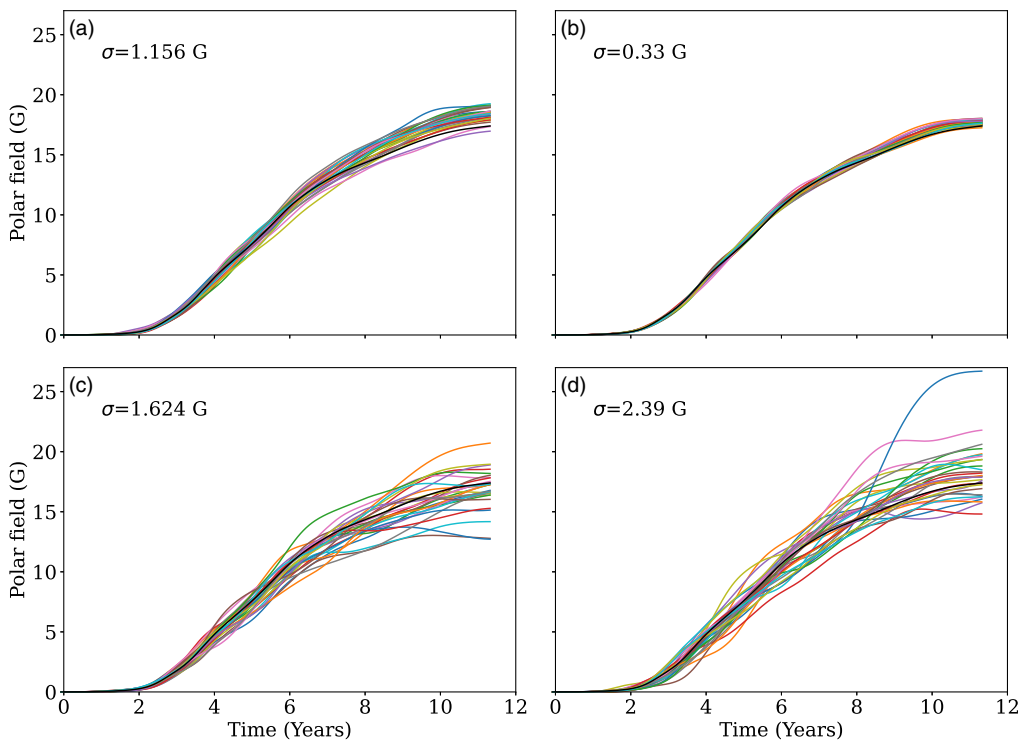

Figure 3. Format of subplots are the same as in Figure 1, but shows the evolution of the polar field from simulations with (a) variation in the BMRs flux, (b) variation in BMRs emergence latitude, (c) scatter in BMR tilt, and (d) combined fluctuations.

higher latitudes (Karak and Miesch 2018; Karak 2020). Moreover, it is important to note that the flux contribution to polar field formation in a given hemisphere is directly proportional to the total active region flux for that hemisphere (Kitchatinov & Olemskoy 2011). Therefore, any changes in the flux can induce variations in the polar field. In addition, the scatter in the tilt arises from wrongly (anti-Joy and anti-Hale) tilted BMRs (Stenflo and Kosovichev 2012). These wrongly tilted BMRs engage in annihilation with
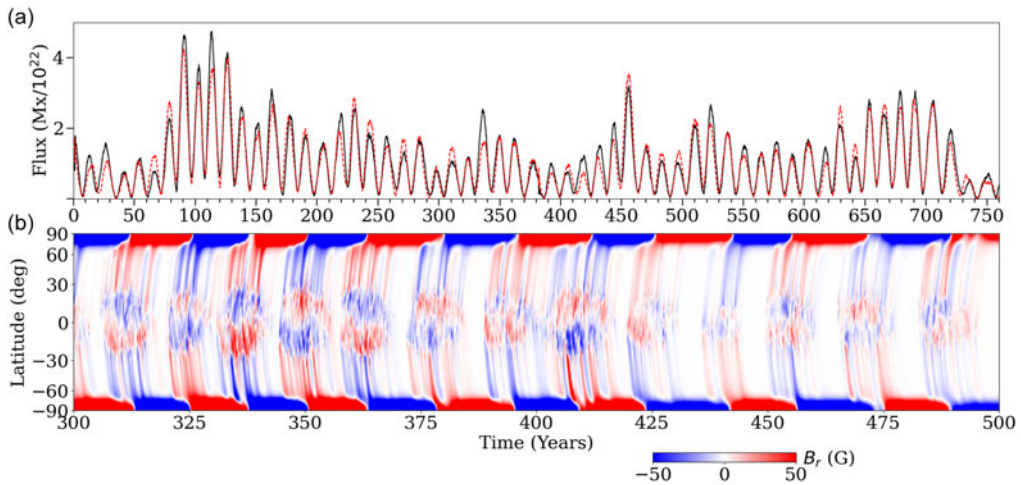
Figure 4. The plot shows the variation in the solar cycle from simulation, including all fluctuations (time delay, scatter in tilt, variation in latitude, and flux). Time delay is considered magnetic field dependent in this simulation.

the normal BMR field, leading to a significant variation in the polar field (Nagy et al. 2017; Golubeva et al. 2023).

3.3. *Variability in the solar cycle*

The variability introduced in the solar cycle due to the irregular properties of BMRs is similar to that observed in the polar field. This result is evident because the polar magnetic field or its proxy strongly correlates with the next cycle's activity level (Schatten et al. 1978; Choudhuri et al. 2007; Kumar et al. 2021b, 2022; Biswas et al. 2023). Similar to the polar field, the impact of time delay on solar cycle variation is found to be negligible. The maximum variability is produced due to the scatter in tilt, wherein BMRs with erroneously oriented Joy's and Hale's laws contribute significantly to fluctuations in the solar cycle. Furthermore, the fluctuations in flux and emergence latitude also contribute substantially to the observed variation in the solar cycle. When all these fluctuations are considered collectively, the resulting variability in the solar cycle aligns consistently with the observed patterns, as depicted in Figure 4.

4. Conclusion

In our investigation, we utilized a 2D and 3D flux transport dynamo model to explore the variability in both the solar cycle and the polar field, considering the observed irregular properties of BMRs and different levels of supercriticality. The simulations revealed that, for the same level of fluctuation, if the dynamo operates near the critical regime, the resulting variation in the solar cycle is large. Conversely, when the dynamo operates near the supercritical regime, the observed variation in the solar cycle is comparatively small. Furthermore, when we consider the irregular properties of BMR, the maximum variation produced in the polar field and the solar cycle is due to scatter in BMR tilt. The variation produced by the time delay is minimal; however, fluctuation in emergence latitude and the BMR flux produces significant variability in the polar field and the solar cycle. Moreover, tilt scatter in BMR tilt, flux variation, latitude variation, and fluctuation in time delay produce variability in the polar field and the solar cycle in the descending

order, respectively. These results also suggest that the irregular BMR properties suffi-ciently produce the variation in the polar field and the solar magnetic cycle, which is consistent with the observations.

5. Acknowledgements

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