Strong Gravitational Lensing in the Era of Big Data
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Accounting for population-level systematic effects using a hierarchical strategy

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Abstract. One of the largest sources of systematics in time-delay cosmography arises from Mass Sheet Transformation (MST). The degeneracy associated with this transformation is often broken by an assumed profile shape, such as a power-law. A hierarchical strategy has been developed which constrains the global profile shape on a population level, constrained collectively by the kinematics measurements of the lenses. This framework allows one to include non-time-delay lenses to provide constraints to the global profile, improving the H_0 constraints. This work tests the hierarchical framework using analytical profiles, and additionally tests the capacity to combine two populations which come from the same profiles but probe different radii due to a change in source redshift. We find that the hierarchical framework is able to compensate for this effect, and the addition of non-time-delay lenses improves the H_0 constraint, even though these lenses have different Einstein radii than their time-delay counterparts.

Keywords. Gravitational lensing: strong, cosmological parameters, Galaxies: statistics

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1. Introduction

This article has been adapted from "TDCOSMO VIII: A key test of systematics in the hierarchical method of time-delay cosmography" (Gomer *et al.* 2022). For more details, we refer the reader to the original work.

The goal of time-delay cosmography is to use gravitational lensing to measure cosmological distances, and hence infer cosmological parameters such as H_0. Typically, the lens is an early-type galaxy and the source is a quasar. For a given system, the time delay between multiple images of the source is measured and compared against that predicted by the lens model, giving a measure of the so-called time-delay distance $D_{\Delta t} \propto H_0^{-1}$. Perhaps the largest source of systematics in such an analysis is due to the Mass Sheet Transformation (MST, Falco *et al.* 1985; Schneider & Sluse 2013):

$$\kappa(\theta) \rightarrow \lambda\kappa(\theta) + (1 - \lambda). \tag{1}$$

For any value of λ, the transformed convergence will reproduce the lensed imaging information and as such λ cannot be constrained by lensing alone. The time delays, and consequently the determination of H_0, are affected by a factor of λ, meaning an accurate determination of H_0 requires a constraint on λ coming from information external to the lensing data. Two sources of such information are possible: an assumed model profile shape (e.g. assumed λ) based on the known structure of early-type galaxies, and/or a measurement of the dynamics of the lens galaxy, which provides a direct measure of mass within some aperture, constraining the physically allowed value of λ. In this work, we restrict ourselves to the internal MST related to profile shape, specified with the notation that $\lambda = \lambda_{\rm int}$.

To date, the most precise measures of H_0 using time-delay cosmography come from the TDCOSMO collaboration (Millon *et al.* 2020). The process consists of jointly modeling lens imaging data and time delays, along with using kinematic measurements to account for the MST. Using a parametric form for the mass profile in the form of a power-law model, H_0 can be inferred to $\sim 2.5\%$ precision by combining the results from 6 systems (Wong *et al.* 2020). However, this result does not completely account for the full flexibility of the MST, as much of the constraint on $\lambda_{\rm int}$ is informed by the power-law assumption. In order to include this systematic effect in the total error budget, it must be accounted for in the modeling procedure.

1.1. *A hierarchical strategy*

Birrer *et al.* (2022) introduced a technique to supplement the parametric method by explicitly folding the effect of the MST into a hierarchical analysis of the lens systems. The approach applies an MST to the lens modeling results on a population level, serving to capture systemic departure from the initial parametric model.

The hierarchical strategy assumes that as a population, the lens galaxies have some global profile shape which is within an MST of a power law. As such, the $\lambda_{\rm int}$ experienced by a given power-law lensing fit is a function of where on this global profile the Einstein radius ($\theta_{\rm Ein}$), lies. Meanwhile, the kinematic measurements of a given system measure the absolute mass within some aperture size, which has the power to constrain $\lambda_{\rm int}$. Therefore, each lens brings a probe of this global mass distribution, with kinematics information at one radius and lensing information at another. The hierarchical framework combines many systems together, allowing for variation within the population, to measure a population-scale $\lambda_{\rm int}$.

More explicitly, this framework ascribes a distribution of MSTs to the galaxy population, where the distribution is described with Gaussian standard deviation $\sigma(\lambda_{\rm int})$. The transformation is allowed to vary as a function of where in the profile the Einstein radius lies, relative to the effective radius of the galaxy light distribution, $\theta_{\rm eff}$. As the exact function of this transformation is unknown, it is approximated as linear with slope α_λ and intercept $\lambda_{\rm int,0}$:

$$\lambda_{\rm int}(\theta_{\rm eff}/\theta_{\rm Ein}) = \lambda_{\rm int,0} + \alpha_\lambda \left(\frac{\theta_{\rm eff}}{\theta_{\rm Ein}} - 1\right), \tag{2}$$

where $\lambda_{\rm int}$ is the internal MST experienced by a given lens. In addition, two more population-scale parameters are included describing the anisotropy of the kinematics of lens galaxies: $\langle a_{\rm ani}\rangle$ and $\sigma(a_{\rm ani})$ describe the mean and Gaussian width of the anisotropy radius of the lens systems, scaled as $a = r_{\rm ani}/r_{\rm eff}$. Applying this framework to 7 TDCOSMO systems, (Birrer *et al.* 2022) found $H_0 = 74.5^{+5.6}_{-6.1}$ km s^{-1} Mpc^{-1}, where the uncertainty now accounts for the MST.

The beauty of this strategy is that its assumptions on the profile shape are data-driven, and as such it can benefit from including additional data. Lens models and kinematics from other systems can be incorporated into the framework to provide better constraints on the profile shape, even if they themselves lack time delays and cannot be used for H_0 determination. The inherent assumption in this practice is that all lens galaxies included in the inference come from the same global population. Birrer *et al.* (2022) combined 33 SLACS lenses with the 7 TDCOSMO results, finding $H_0 = 67.4^{+4.1}_{-3.2}$ km s^{-1} Mpc^{-1}.

The results are consistent with one another, although the mean value has shifted. The main concern with this analysis is that if the SLACS systems are systematically different than the TDCOSMO systems, combining them could bias the resulting value of H_0. The most obvious difference between these systems is that they have different redshifts and therefore probe different parts of the profile. In theory, the linear scaling with radius should account for this change.

This sets the stage for a targeted test of systematics in the hierarchical strategy. In this work, we will use mock lenses within the same population for which different parts of the profile are probed by lensing due only to a change in redshift. Modeling and combining these systems hierarchically provides a test to see if the hierarchical framework can account for this effect.

2. Experiment setup

We test the hierarchical framework using a population of mock lenses with analytical mass distributions. This mock population resembles the SLACS lenses (Shajib *et al.* 2021) when the source is placed at a redshift of $z_s = 0.6$, and resemble the TDCOSMO lenses when the source is placed at a redshift of $z_s = 2.0$. These choices result in critical densities for each set which is consistent with its respective population.

Each lens is described using two components: a Chameleon profile emulates a Sérsic profile to represent the stellar mass distribution, while an NFW component represents the dark matter distribution. To determine the parameters for the Chameleon and NFW profiles, we sample a large range of values and select the resulting profiles which match the observed populations by several metrics, namely the Einstein radius $\theta_{\rm Ein}$, when placed at both $z_s = 0.6$ and $z_s = 2.0$, the effective radius of the light distribution $\theta_{\rm eff}$, and the axis ratio q. Selecting systems which match these criteria give us a set of systems for which we can create a "SLACS-like" system when placing the source at $z_s = 0.6$, and a "TDCOSMO-like" counterpart for which the only change is that the source is now at $z_s = 2.0$.

We randomly select 20 pairs of "SLACS-like" and "TDCOSMO-like" mock profiles to test the hierarchical framework. First, we create mock images from these profiles using `lenstronomy` (Birrer *et al.* 2018, 2021a), with resolution and noise settings matching the F160W filter on the HST WFC3 camera. The mock source is a circular Sérsic source, and the mock lens light is not included so as to emulate a perfect light subtraction. The sources for the TDCOSMO-like systems include a point source which emulates a quasar. The TDCOSMO-like systems are also given mock time-delay measurements. Each mock is fit using Power-Law Elliptical Mass Distribution (PEMD) + external shear model using `lenstronomy`. All mock images are successfully fit to the level of the noise, with a reduced $\chi^2 < 1.06$ in all cases.

Mock kinematics are calculated using spherical Jeans kinematics, in which the velocity dispersion is calculated within an aperture under the assumption of a relaxed system with spherical symmetry. Mock kinematic measurements are calculated for three different apertures within 2" and given an uncertainty of 10%, emulating spatially-resolved kinematics observations. For the truth anisotropy, we set $a_{\rm ani} = 1$.

The systems are then combined hierarchically, using the `hierArc`† package. Input into the framework are the posteriors from the lens modeling as well as the kinematics measurements. The framework then recovers the population-scale $\lambda_{\rm int}$ according to Eq. 2. Aperture kinematics scale with $\sqrt{\lambda_{\rm int}}$, thereby informing the nature of the transformation from the power-law models to the global profile shape. Ultimately, posteriors are recovered which explicitly include the effect of $\lambda_{\rm int}$ in the error budget of H_0.

3. Results and Discussion

Before combining the systems hierarchically, the individual power-law fits recover values of H_0 which are biased by approximately 15%, due to the MST. For this work, we combined hierarchically the TDCOSMO-like lenses alone, the SLACS-like lenses alone,

† https://github.com/sibirrer/hierArc

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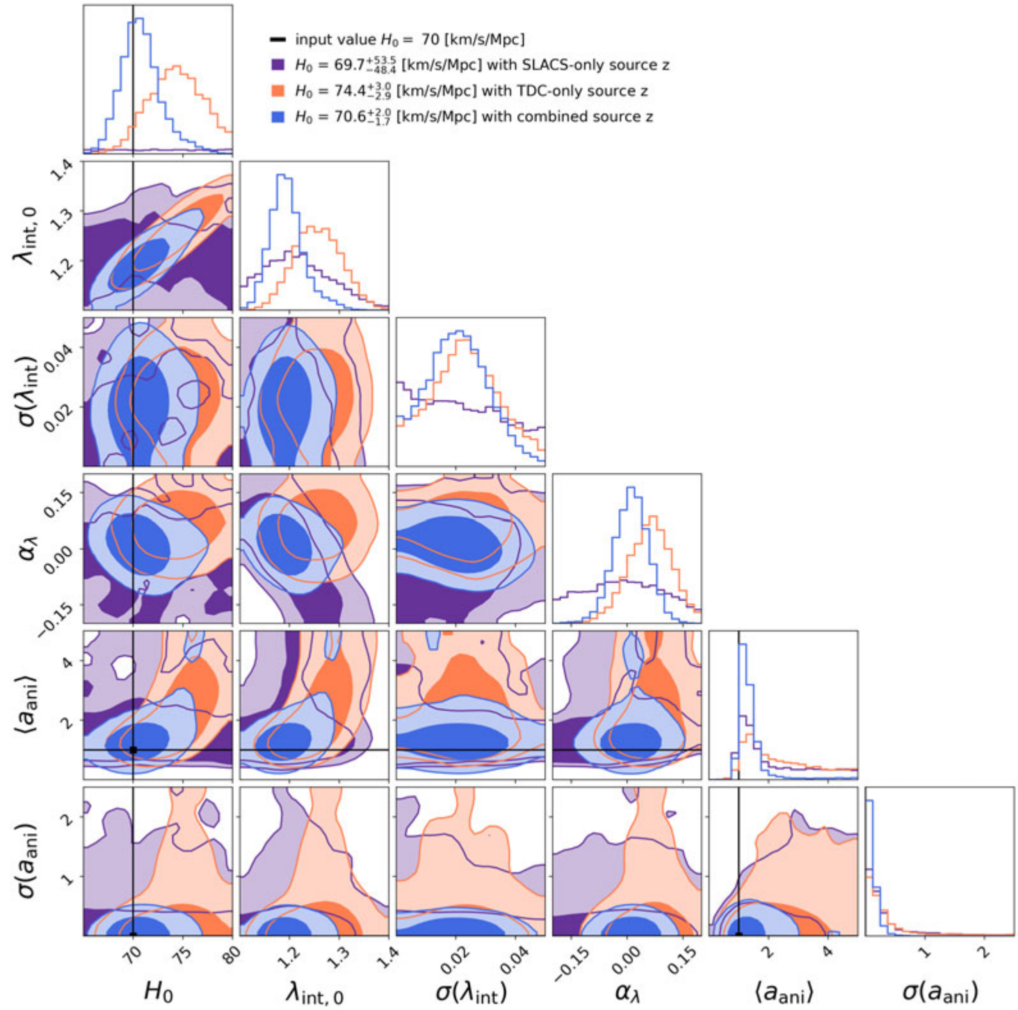

Figure 1. Corner plot for the results of the hierarchical test. Orange: TDCOSMO-like lens population; Purple: SLACS-like population; Blue: Both populations combined. Credit: Gomer *et al.* (2022).

and the combination of the TDCOSMO-like and SLACS-like systems together. These results are shown in Fig. 1.

The TDCOSMO-only result recovers an H_0 of $74.4^{+3.0}_{-2.9}$ km s^{-1} Mpc^{-1}. This posterior is within 1.5σ of the fiducial value of 70 km s^{-1} Mpc^{-1}, with the median now within 6% of the fiducial value. This improvement in accuracy compared to the nonhierarchical case comes from the explicit treatment of $\lambda_{\rm int}$, which is recovered to have a value different than 1.

The result from the SLACS only lens puts constraints on parameters such as $\lambda_{\rm int}$ and $\langle a_{\rm ani}\rangle$, but since the SLACS-like systems lack time delays they cannot constrain H_0 alone. However, the combination of the two populations hierarchically provides the strongest result, recovering H_0 of $70.6^{+2.0}_{-1.7}$ km s^{-1} Mpc^{-1}, closely matching the fiducial value. The additional constraints on the parameters other than H_0 informs the population-scale MST mapping, allowing an accurate recovery of H_0.

Additional tests with a larger sample size found that this precision continues to increase with more systems. This means that the intrinsic scatter in $\lambda_{\rm int}$ is captured by the hierarchical framework and does not pose a systematic floor to the achievable H_0 precision.

3.1. *A look ahead for hierarchical strategies*

The hierarchical framework has been shown to adequately account for the most well-studied lensing degeneracy: the MST. However, there are other types of degeneracies that can be a nuisance in lens modeling. Van de Vyvere *et al.* (2022a,b) explored the effect of azimuthal structure on time-delay cosmography, finding that structures such as disciness, boxiness, and ellipticity gradients can bias the recovered value of H_0 for individual systems, although as a population this effect seems to average out.

We have seen that a hierarchical strategy can allow one to probe the structure beneath the statistical scatter by considering a population, and so naturally one wonders if this could be an applicable strategy to approach this problem. The main challenge is to express an analytical description of these complex degeneracies, which would map a given azimuthal perturbation to the effect it has on H_0 for an individual system. Such a description could then be applied hierarchically to a population, recovering the expected azimuthal contribution on a population scale. This is an ongoing field for exploration for lensing theory (see also Sonnenfeld 2017; Wagner 2019; Kochanek 2021; Birrer 2021b; Gomer *et al.* 2023). Despite its theoretical challenges, such a method may be the way forward for time-delay cosmography in the future, particularly in the big data era where there are too many lenses to model in great detail.

4. Conclusion

Perhaps the largest source of systematic errors in time-delay cosmography comes from an uncertainty on the profile shape, through the Mass Sheet Transformation (MST). A hierarchical framework has been implemented which accounts for the MST on a population level, constrained by the kinematics of many lens systems. In this work, we test this hierarchical framework using analytical profiles, and find that it reduces the bias associated with the MST from $\sim 15\%$ to $\sim 6\%$.

The strength of the hierarchical strategy is that additional systems can be added to inform the MST at the population level, even if they do not directly constrain H_0 alone. Birrer *et al.* (2022) combined SLACS and TDCOSMO systems together using this framework, although one possible concern is that if these systems come from different populations, this could return a biased result. We perform a targeted systematics test by creating mock SLACS-like lenses which have the same profiles as their TDCOSMO-like counterparts, except that a change in source redshift changes the Einstein radius. We show that the hierarchical framework is able to accommodate this systematic change, and that the inclusion of these SLACS-like systems results in a more accurate H_0. We note however that any evolution in the global properties of the profile has not been tested by this work: to capture such trends may require that a parameterization of such trends be included in an updated hierarchical implementation. Nevertheless, this result strengthens the argument for a hierarchical strategy, which we speculate will become a predominant method for time-delay cosmography in the era of big data.

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