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(L^p, L^q) -multiplier problems

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Let A and B denote two linear topological vector spaces of functions, measures or distributions over a locally compact group which are invariant under all left translation operators. A continuous linear operator from A into B is called a *multiplier*, or an (A, B)-*multiplier*, if it commutes with left translations. The thesis is essentially a study of (L^p, L^q)-multipliers, the types of questions studied being largely influenced by the structure of the underlying group in each case. Much of the interest in (L^p, L^q)-multipliers stems from the fact that when the underlying group is the circle group there is a direct relation with the classical theory of factor functions, namely: "A continuous linear operator T from L^p, $p \neq \infty$, into L^q is a multiplier if and only if there exists a sequence $(\mu_n)_{n=-\infty}^{n=\infty}$, a factor function, such that $\sum_{n=-\infty}^{n=\infty} \mu_n a_n e_n$ is the Fourier series of Tf whenever $\sum_{n=-\infty}^{n=\infty} a_n e_n$ is the Fourier series of f in L^p, where e_n denotes the function $x \mapsto e^{inx}$".

One of the new results obtained is that if the underlying group is infinite compact or infinite locally compact abelian, then for p satisfying $1 < p < 2$,

$$\bigcup_{1 \le q < p} L^q_q \subsetneq L^p_p \subsetneq \bigcap_{p < q \le 2} L^q_q ,$$

where L^p_p denotes the set of (L^p, L^p)-multipliers restricted to any set

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dense in L^p for all p satisfying $1 \leq p < \infty$, for example, restricted to the set of continuous functions with compact supports. Other problems tackled are indicated in the following list of chapter headings:

Chapter 1, Multipliers with range in the space of temperate distributions;

Chapter 2, Multipliers between some normed spaces of distributions;

Chapter 3, Representations of (L^p, L^q)-multipliers when G is compact;

Chapter 4, Idempotent multipliers and lacunary subsets of the dual of G;

Chapter 5, Complemented closed ideals in L^p;

Chapter 6, Multipliers which are not measures;

Chapter 7, The strict inclusion $L^p_p \subsetneq L^q_q$;

Appendix A, Some boundedness theorems;

Appendix B, (L^∞, L^q)-multipliers when L^∞ has its weak topology;

Appendix C, A constructive approach to boundedness principles.

Apart from several minor results, the original work of the thesis has been prepared for publication and has either appeared recently or will appear shortly.

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