PART III

ASTEROIDS

https://doi.org/10.1017/S0252921100097001 Published online by Cambridge University Press

FAMILIES OF ASTEROIDS

Yoshihide Kozai

Tokyo Astronomical Observatory, Mitaka, Tokyo 181, Japan

The numbered asteroids are classified into families by a new method proposed by the author(Kozai, 1979a) and their characteristics are studied by several aspects. It seems to the author that there are two kinds of the families, one being very compact in the phase space and containing many faint as well as a few bright asteroids and one being rather loose and bounded by secular and mean motion commensurable regions.

Families of asteroids were discovered by Hirayama(1918) by finding several clusterings of the asteroids with similar osculating values of the semi-major axis, a, the eccentricity, e, and the inclination, i. He then showed that the proper eccentricity and inclination of each member in the same clustering take values more similar to each other. As the semi-major axis as well as the proper elements are stable quantities in the theory of the secular perturbations of the asteroids, he thought that the asteroids in the same clustering have a common origin, and, therefore, the name of the family was given to the clusterings. Later Brouwer(1951) and others(Arnold, 1969; Williams, 1971 and 1979; Lindblad and Southworth, 1971) restudied the family of the asteroids. Except for Williams they tried to group the asteroids into families by using the proper elements which are based on the classical theory of the secular perturbations with the semi-major axis. Williams used his own theory to compute the proper elements.

In the classical theory the eccentricity and the inclination of any of the asteroids as well as those of the disturbing planets are assumed to be small quantities of the order of the square root of the disturbing mass, namely the mass of Jupiter and the terms with factors of the squares of the disturbing masses are neglected in the disturbing function and the orbital elements of the disturbing planets are assumed to be known functions of time, namely the expressions obtained by the classical secular perturbation theory of the major planets. Then the equations of motion are reduced to two independent sets of linear differential equations, one for the eccentricity and the longitude of the perihelion and one for the inclination and the longitude of the node, and the solutions are expressed by the sums of free oscillation and forced oscillation terms which are

V. V. Markellos and Y. Kozai (eds.), Dynamical Trapping and Evolution in the Solar System, 117–122.

produced by the secular perturbation terms in the expressions of the orbital elements of the disturbing planets. The frequencies of the free oscillations for the two sets are the same in the absolute value but opposite in the sense, the sum of the two frequencies vanishing. The proper frequencies represent for most of the cases the secular motions of the longitudes of the perihelion and of the node, respectively, and depend on the semi-major axis of the asteroid only for the linear theory. The amplitudes of the free oscillations are called the proper eccentricity and inclination which are constant in the linear theory.

However, it is evident that when higher order and degree terms are included in the disturbing function the equations of motion for the secular perturbations are no more linear and the sum of the two secular motions does not generally vanish. Since there are many asteroids, for which the eccentricity and/or inclination are as large as 0.1 to 0.2 the effects of the neglected terms in the linear theory are not so small that they cannot be regarded as small corrections for some cases.

The author(Kozai, 1962) showed that when the eccentricity and/or inclination are not small they change with the argument of perihelion appreciably due to the neglected terms in the disturbing function in the linear theory and the amplitudes of the variations are for some cases larger than those of the forced oscillations. The secular motions depend also strongly on the squares of the eccentricity and the inclination. There are some asteroids, for which the arguments of perihelion do not complete revolutions but librate around 90° or 270°, which is not possible in the classical linear theory.

When it can be assumed that the disturbing planets move along circular orbits on the same plane, the value of $[a(1 - e^2)]^{1/2}\cos i$, the z-component of the angular momentum density, is constant. After averaging the equations of motion or the Hamiltonian with respect to the mean longitudes of the asteroid and the disturbing planets the energy integral can be obtained, namely, the Hamiltonian is constant then since the time does not appear explicitly there. The averaging with respect to the longitudes can be done when there is no commensurable relation easily. Since the semimajor axis is constant in the averaged system, the system of the equations is now reduced to that of one degree of freedom with the energy integral, and, therefore, it can be solved by a quadrature mathematically.

The equations of motion show that the value of $(1 - e^2)^{1/2}$ is a periodic function of twice the argument of perihelion. Since $\Theta = (1 - e^2)^{1/2}\cos i$ is constant both the eccentricity and the inclination are also periodic functions of twice the argument of perihelion. The eccentricity is minimum and the inclination is maximum when the argument of perihelion is 0° and 180° and vice versa when it is 90° and 270°. However, when the argument of perihelion librates around 90° or 270°, both the maximum and the minimum of the eccentricity and the inclination take place at 90° or 270° which the argument of perihelion takes twice in one period. The inclination in this paper is referred to the orbital plane of Jupiter which is also assumed to be the common plane for all the disturbing planets.

For all the numbered asteroids the maximum and the minimum values of the eccentricity and the inclination as functions of the argument of perihelion are computed and stored in a computor file. In one of the previous papers(Kozai, 1979a) it was proposed that the semi-major axis, the minimum value of the inclination, i_{min}, and Θ be used as the three parameters to classify the asteroids into families. They are more adequate than the semi-major axis and the two proper elements based on the linear theory, particularly,when the eccentricity and/or the inclination are not small. In fact there are several such families.

It is, again, tried to find several clusterings in the three-dimensional phase space of a, i_{min} and Θ for 2 700 numbered asteroids. In Figure 1 distributions of i_{min} with respect to the semi-major axis are shown. In the figure several clusterings clearly show up. In Table 1 data for nine major families which were found by Hirayama(1918 and 1923) are given. The names of the families were also given by Hirayama. However, Flora family which is originally one family(Hirayama, 1923) is divided here into three families as the original one by Hirayama is very wide.

Table 1. Data for Nine Major Families

Family	Number	a(AU)	Θ	i_{min}	Hirayama	Brouwer
Flora I	66	2.16 - 2.28	0.992 - 0.999	0°.5 - 5°.5		
II	155	2.16 - 2.30	0.983 - 0.992	0.6 - 7.5	63	125
III	143	2.18 - 2.30	0.954 - 0.983	2.0 - 9.6		
Phocaea	52	2.30 - 2.43	0.850 - 0.916	20.3 - 26.0	11	21
Maria	42	2.52 - 2.58	0.938 - 0.975	12.1 - 15.9	14	17
Pallas	9	2.62 - 2.79	0.794 - 0.827	27.9 - 30.6	3	6
Coronis	76	2.83 - 2.91	0.994 - 1.000	1.7 - 2.5	20	33
Eos	160	2.97 - 3.07	0.973 - 0.989	8.0 - 11.3	27	38
Themis	157	3.07 - 3.22	0.976 - 1.000	0.6 - 2.5	32	53

In the column under the heading "Number" the numbers of the member asteroids are given, whereas under the headings "Hirayama" and "Brouwer" the numbers in their works are written. It is clear that the numbers of the members have been increased very much as Hirayama and Brouwer used, respectively, about 950 and 1540 asteroids for their family works.

In the Table 1 the ranges of the semi-major axis, i_{min} and Θ are given for each family and it is clear that the families of Coronis, Eos and Themis are very compact clusterings in the phase space with defined boundaries whereas those of Flora, Phocaea, Maria and Pallas are not. However, for Pallas family, for an example, the density in the phase space is much higher in the family than that in the surroundings since there are only few asteroids with so large inclinations and eccentricities as those of Pallas family asteroids generally. Therefore, the clustering for Pallas family is very remarkable in the phase space. The situations for Flora families are a little different, as the inclinations and the eccentricities for the asteroids in Flora families are rather small. However,there are so many asteroids in Flora families to make very big clusterings.

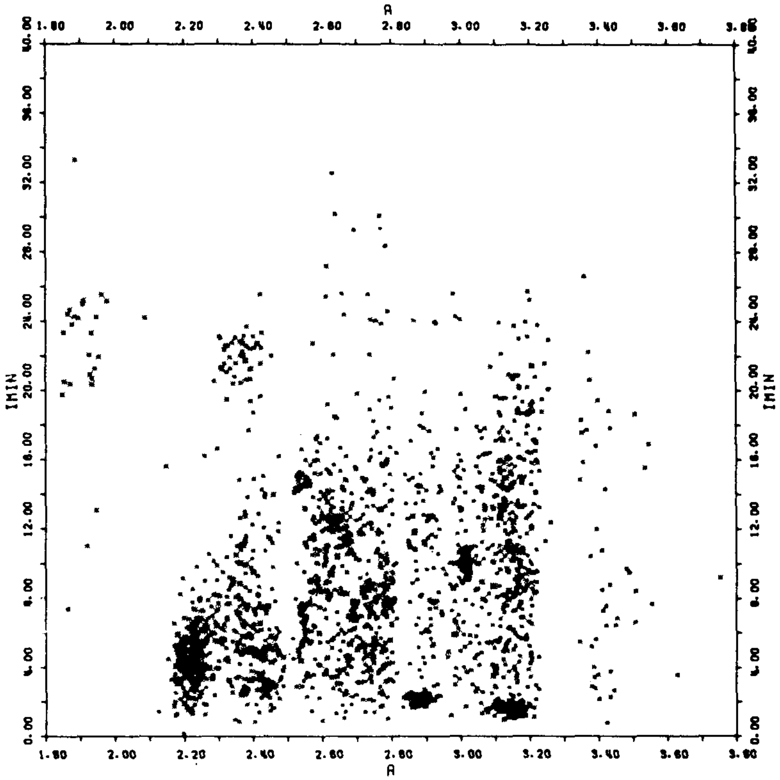

Figure 1. Distribution of i_{min} with respect to the semi-major axis for the numbered asteroids.

In Table 2 the numbers of the member asteroids are given for the seven groups according to their serial registration numbers, the number regions being 1-450, 451-900, 901-1350, 1351-1800, 1801-2250 and 2251-2700.

Table 2. Numbers of the Member Asteroids in Each Family according to the Serial Registration Numbers.

Family	1 - 450	451 - 900	901 -1350	1351-1800	1801-2250	2251-2700
Flora I	6	9	2	15	12	20
II	12	17	23	36	26	40
III	8	14	30	35	32	24
Phocaea	7	3	9	10	17	6
Maria	6	15	7	5	3	6
Pallas	1	2	3	1	1	1
Coronis	8	8	11	16	22	11
Eos	10	26	25	32	31	35
Themis	15	16	23	24	35	43
Total	73	110	133	174	179	186

Table 2 shows that compact and/or large families contain much more faint and small asteroids rather than bright and large ones as asteroids with large serial registration numbers are generally fainter as they could not be discovered by using small telescopes in old days. This tendency is remarkable for Eos, Themis and Flora families as about two fifths of the asteroids numbered in 2251-2700 belong to these families. This may suggest that in such compact and/or large families certainly some kinds of fragmentations of asteroids took place very frequently while in small families like Pallas, Phocaea and Maria fragmentations did not take place so frequently as there are not many faint asteroids there.

It is also true that every major family is near resonant regions. In fact it is possible to find some commensurable ratios for the mean motions with Jupiter corresponding to the boundaries of the families as it is shown in Table 3.

Table 3. Relations between Ratios of the Mean Motions with Jupiter and Boundaries of Families.

Ratio	15:4	17:5	19:6	3:1	17:6	5:2	12:5	7:3	11:5	2:1
a(AU)	2.16	2.30	2.41	2.50	2.60	2.82	2.90	2.96	3.07	3.28
Family	Flora		Phocaea		Maria	Pallas	Coronis		Eos	Themis

Also some of the boundaries in the phase space correspond to secular commensurable regions(Williams and Faukner, 1981). It seems to the author, therefore, that such commensurable regions did have some effects for the fragmentations of asteroids in the places of the families as it is suspected that the motions might be unstable there. In fact an idea of the author on the origin of the families is that originally several regions with high number density of asteroids were created by the resonances of the mean motions and secular motions and since there were much more asteroids collisions to create families took place more frequently than other regions.

Besides the nine families shown here many other families can be found as there are other clusterings in Figure 1. However, for other possible families clear boundaries such as those for the compact families cannot be found. According to criteria assigned to classify the asteroids different families may be found. Also it is necessary to depend on more exact theory of the secular perturbations as well as periodic ones to identify more families. As it was mentioned earlier many families are situated near commensurable regions, either for the mean or secular motions. Therefore, it is very difficult to develop any adequate theory which is valid for such regions. In fact the discussions about the origin of the families need more advanced theories on the motions of asteroids which will be the next important target of celestial mechanics. And any study on the families will contribute much to researches on origins of the asteroids and the solar system.

The computations in this paper were made by using the UNIVAC 1100/80B computer at the Tokyo Astronomical Observatory.

REFERENCES.

Arnold, J.R.: 1969, "Astron. J." 74, p.1235.
Brouwer, D.: 1951, "Astron. J." 56, p.9.
Hirayama, K.: 1918, "Astron. J." 31, p.185.
Hirayama, K.: 1923, "Japan J. Astron. Geophys." 1, p.55.
Hirayama, K.: 1928, "Japan J. Astron. Geophys." 5, p.137.
Kozai, Y. : 1962, "Astron. J." 67, p.591.
Kozai, Y. : 1979a, In "Dynamics of the Solar System" ed. by R.L.Duncombe, Reidel, p.231.
Kozai, Y. : 1979b, In "Minor Planets" ed. by T. Gehrels, University of Arizona Press, p.334.
Lindblad, B.A. and Southworth, R.B.: 1971, In "Physical Studies of Minor Planets" ed. by T. Gehrels(NASA SP-267), p.337.
Williams, J. G. : 1971, In "Physical Studies of Minor Planets" ed. by T. Gehrels(NASA SP-267), p.177.
Williams, J.G.: 1979, In "Minor Planets" ed. by T. Gehrels, University of Arizona Press, p.1040.
Williams, J.G. and Faukner, J.: 1981, "Icarus" 46, p.390.