CORRESPONDENCE.

LIFE-CONTINGENCY PROBLEMS.

To the Editors of the Assurance Magazine.

GENTLEMEN,—I now send you my solutions of the three Problems which I proposed in Number II. of the Magazine.

Annuities.—Taking that form of the Column System in which $N \div D = (1+A)$, *i. e.* the value of a pre-annuity, or an annuity payable in advance, or at the *beginning* of each year, then $N_1 \div D = A$, or the ordinary annuity, may be considered as a pre-annuity *deferred* one year. In the same manner, an annuity payable by two equal instalments during the year may be considered as two deferred pre-annuities,—the one deferred six months, the other deferred twelve months; and (*generaliter*) an annuity payable by n instalments during the year may be considered as n separate pre-annuities severally deferred one, two . . . . . . n parts of the year. Hence, the formula deducible, according to the Column System, for an annuity payable by n instalments during the year would be $(N_{1.n} + N_{2:n} + N_{3:n} \ldots\ldots N) \div nD$. In place therefore of interpolating the original Table of Mortality, it will be sufficient to interpolate the values of N from N to N_1; and it is obvious this can be done to any degree of practical precision by aid of the calculus of Finite Differences, based on a sufficient number of collateral terms.

The ordinary and useful formula of adjustment by which an annuity payable by n instalments during the year is considered as increased in value by $\frac{n-1}{2n}$ parts of a year's purchase, we virtually owe to Simpson ("Annuities," p. 79): but its original demonstration, and the extended one inserted in Milne's (Art. 455–485) and other treatises, being rather vague, it is among the advantages of the Column System that it at once places the eligibility of this approximation in a clear light; for we have only to suppose the interpolations from N to N_1 to be in arithmetical progression, to arrive at a similar formula. Thus, remembering that an ordinary annuity *increased* by $\frac{n-1}{2n}$ parts, is the same as a pre-annuity *decreased* by $\frac{n+1}{2n}$ parts, and that $N - N_1 = D$, we have $\frac{D}{n} = \Delta$ for the common difference of interpolation: hence the value of the annuity payable by instalments will, on this supposition, be

$$[(N-\Delta)+(N-2\Delta)+(N-3\Delta)\ldots\ldots\ldots+(N-n\Delta) \text{ or } N_1] \div nD,$$

which equals $\left(nN-(n+1)\frac{n}{2}\Delta\right)\div nD=(1+A)-\frac{n+1}{2n}$; for $n\Delta=D$.

Assurances.—The present value of any sum assured, payable with interest upon the death of A, is obviously equal to the sum assured itself, for no other sum will exactly produce a similar amount at interest. If then we put P as the single premium to secure £1 at death, $(1-P)$ is obviously the present value of the interest of the sum assured considered as a loan; and the value of the policy after n years will be found to be equal to the difference between the sum assured and the relation of the value of the original interest to that commencing at the later period, or the value of the policy will equal $1-\frac{(1-P_n)}{(1-P)}$. For $(1-P_n)=(1-v)\,(1+A_n)$, and $(1-P)=(1-v)\,(1+A)$: hence $1-\frac{(1-P_n)}{1-P}=1-\frac{1+A_n}{1+A}$, which is Mr. Griffith Davies' well-known formula for the value of a policy. We can also put $1-\frac{(1-P_n)}{(1-P)}$ into the form of $\frac{(1-P)-(1-P_n)}{1-P}=\frac{(P_n-P)}{(1-P)}$; or the theoretical value of a policy when a premium is due is equal to the difference between the two single premiums divided by the difference between the original single premium and the sum assured. It is also obvious from the preceding deductions, that the same relation obtains when stated in similar terms of the present value of the interest of the sum assured considered as a loan at the two periods; the increase of the single premium being exactly equal to the decrease in the value of the future interest.

Probabilities.—There may be said to be two principal methods of considering a "definite integral:" the one which regards it as a primitive function from which a certain differential co-efficient can be derived; the other which treats it as the determination of the limit to which the summation of an infinite series of functional values tends, when each term of the series is to be comprised between certain terminal values, and is, moreover, to be multiplied by the limit of magnitude to which the greater and greater subdivision of the difference between the limits of the variable ultimately points. Now, it is evident, that if the difference, as in the case proposed, between the limits of the variable be unity, that this common factor may be represented by $\frac{1}{n+1}$. The effect, therefore, is really equivalent to taking a certain proportion only of each term; and this effect is precisely that which is indicated when an average has to be taken, provided the proportion correspond to the number of terms, as it obviously does in a definite integral whose limits are zero and unity: for as n increases, the limiting ratio of $\frac{1}{n}$ to $\frac{1}{n+1}$, or $1 : \frac{1}{1+\frac{1}{n}}$, becomes more and more equal to unity. Now, whatever law of facility of error, or of deviation among a set of observations be supposed, it has been well shown by Professor De Morgan, (*Ency. Metrop.*, art. "Probab.,") that the average term and the most probable value approach nearer and nearer to an equality as the number of data or values increases; and this is precisely the same condition as that under which the value or summation of the definite integral more and more accurately represents the limiting value of the average term. It may also be seen, by reference to an article in the July number of the *Edinburgh Review* (No. 185, p. 19,) on Probabilities, said to be by Sir John Herschel, that the same conditions, above declared to be inherent in definite integration, and therefore in averaging upon the system of limits, have to be also stipulated for in the postulates, whenever the law of the results has to be determined in its utmost generality.

E. J. F.

[NOTE.—We have received from Mr. William Wylie, of the Colonial Life Assurance Company's Office, in Edinburgh, ingenious solutions of the first and third of these Problems.—ED. *A. M.*]

ON THE DETERMINATION OF SURPLUS.

To the Editors of the Assurance Magazine.

GENTLEMEN,—I have been very much gratified with the article in the *Assurance Magazine* on the Determination of the Surplus of a Life Assurance Company. It may perhaps interest some one to see the process which I have used for the same purpose.

It should be premised that it is the practice in the American Companies to assure at the age of the nearest birthday, so that no material error can arise from assuming the day of the date of the policy as the birthday of the party assured.

In the first place, I arrange the policies according to the year of birth, as in the article referred to, but grouping them according to the age at which they were assured, and the consequent premium paid: thus—