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The conjugate of a smooth Banach space

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A Banach space X is *smooth* if at every point of the unit sphere there is only one supporting hyperplane of the unit ball; and *strictly convex*, or *rotund*, if the unit sphere contains no line segment.

Although there is a strong duality between these notions, Klee has produced a smooth space whose conjugate is not rotund. However there is no known example of a smooth space with conjugate not isomorphic to a rotund space.

The main purpose of this note is to show that if X is a smooth space with a certain property, X^* is isomorphic to a rotund space. This will follow from a mapping theorem which implies the existence of a set Γ and a continuous one-to-one linear map T of X^* into $c_0(\Gamma)$.

1. Introduction and summary

Throughout this paper we assume X to be a real infinite dimensional Banach space with X^* and X^{**} denoting its first and second conjugate spaces respectively. If x is an element of X we denote by $\hat{x}$ the element of X^{**} defined by $\hat{x}(f) = f(x)$ for $f \in X^*$. If X is smooth for $x \in X$ we denote by f_x the unique element of X^* such that $\|f_x\| = \|x\|$ and $f_x(x) = \|f_x\|\|x\|$. It is well known [3, p. 300] that if X is smooth and $x_n \to x$ in the norm topology then $f_{x_n} \to f_x$ in the

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weak* topology. We say that a Banach space has *property* A if it is smooth and if, whenever $x_n \to x$ in norm, $f_{x_n} \to f_x$ in the weak topology. In particular it follows that strongly smooth spaces have property A [9, p. 140], as do smooth Grothendieck spaces. However, not all smooth spaces have property A, as can be seen from Lemma 6.

The main result of this note is the following mapping theorem:

THEOREM 1. *Let* X *be a Banach space with property* A. *Then there exist a set* Γ *and a bounded one-to-one linear map* T *from* X^* *into* $c_0(\Gamma)$.

We recall at this point that $c_0(\Gamma)$ is the Banach space consisting of the real-valued functions f on Γ which vanish at infinity; i.e., such that $\{\gamma : \gamma \in \Gamma, |f(\gamma)| > \varepsilon\}$ is finite for every $\varepsilon > 0$.

The theorem should be compared with the following powerful theorem of Lindenstrauss [11]: *If* X *is a reflexive Banach space, then there exist a set* Γ *and a continuous one-to-one linear map* T *of* X *into* $c_0(\Gamma)$. In fact Theorem 1 follows from this result if we assume X to be a conjugate space, for then X is reflexive by a generalisation of a result of Šmulian (see, for example, [7, Theorem 2]). More generally Amir and Lindenstrauss [1] have shown that if X is the closed linear span of a weakly compact subset of X, then there exist such a set Γ and mapping T.

We prove our other stated result as a corollary to Theorem 1 at this point.

COROLLARY. *Let* X *be a Banach space with property* A. *Then* X^* *is isomorphic to a rotund space.*

Proof. By the main theorem there exist a set Γ and a one-to-one bounded linear map $T : X^* \to c_0(\Gamma)$. Now by Day [5, p. 523] $c_0(\Gamma)$ admits an equivalent strictly convex norm $|\cdot|$. We renorm X^* by putting $|f| = \|f\| + |Tf|$. It is readily checked that $|\cdot|$ is an equivalent strictly convex norm on X^* and so the result follows.

We comment that this clarifies a point made in Day [5, p. 518] and

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Cudia [4, p. 88]. We point out that though we consider spaces over the reals the proofs need only slight modifications in the complex case.

2. Proof of Theorem 1

The proof is based on techniques developed by Lindenstrauss [10 and 11]. It is long and is broken up by a series of lemmas.

The first result is due to Lindenstrauss [11, p. 967].

LEMMA 1. *Let* X *be a Banach space and let* B *be a finite dimensional subspace of* X . *Let* k *be an integer and suppose* $\varepsilon > 0$. *Then there is a finite dimensional subspace* Z *of* X *containing* B *such that for every subspace* Y *of* X *containing* B *with* $\dim Y/B = k$ *there is a linear operator* $T : Y \to Z$ *with* $\|T\| \le 1+\varepsilon$, *and* $Tb = b$ *for all* $b \in B$.

We denote by X^α the space of homogeneous functionals on X which are bounded on the unit ball of X . For $f \in X^\alpha$ we define a norm by $\|f\| = \sup\{|f(x)| : \|x\| = 1\}$. It is easily seen by a slight generalisation of the Banach-Alaoglu theorem, or by a direct application of Tychonoff's theorem, that the unit ball of X^α is compact in the $\hat{X}$-topology. If T is a map from C^* into X^* , where C is a subspace of X , we denote by $\tilde{T}$ the extension map of T from X^* into X^* defined by $\tilde{T}(f) = T(\underline{f})$, where $\underline{f}$ is f restricted to C . We retain this notation for the remaining lemmas.

LEMMA 2. *Let* X *be a Banach space and let* B *be a finite dimensional subspace of* X . *Then there exist a separable subspace* C *of* X *and a linear operator* $T : C^* \to X^*$ *such that* $\|T\| = 1$ *and* $\tilde{T}^*\hat{x} = \hat{x}$ *for all* $x \in B$.

Proof. Let $C_n \supset B$, $n = 1, 2, \ldots$ be the subspaces of X given by Lemma 1 for $k = n$, $\varepsilon = 1/n$, and let $C = \overline{sp}\left(\bigcup_{n=1}^{\infty} C_n\right)$. If E is a subspace of X containing B , such that $\dim E/B = n$, then there is a linear operator $T_E : E \to C$ such that $\|T_E\| \le 1 + 1/n$, $T_E x = x$ for all $x \in B$. We extend T_E to a map (not linear) $T'_E : X \to C$ by

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defining $T'_E x = 0$ if $x \in X \backslash E$.

We consider the adjoint map $T'^*_E : C^* \to X^\alpha$. In the space of all bounded linear maps $C^* \to X^\alpha$ we take the pointwise topology, and on X^α the $\hat{X}$-topology. As the unit ball of X^α is $\hat{X}$-compact, Tychonoff's theorem ensures that the net $\{T'^*_E : E \supset B\}$ (here we order the subspaces E by inclusion) has a limit point $T : C^* \to X^\alpha$.

It is straightforward to check that $T : C^* \to X^*$, and that it satisfies the conditions of the lemma.

If Y is a closed subspace of X we denote by $D_{X^*}(Y)$ the set of $f \in X^*$ which attain their norm on the unit sphere of Y . If $D_{X^*}(X)$ is norm dense in X^* , X is said to be subreflexive. E. Bishop and R.R. Phelps [2] have shown that all Banach spaces are subreflexive. We couple this result with smoothness to obtain:

LEMMA 3. *Let X be a smooth space, let x_i , $i = 1, \ldots, n$, and f_j , $j = 1, \ldots, m$, be finite sets in X and X^* respectively, and let $\varepsilon > 0$. Then there exist a separable subspace C of X and a linear operator $T : C^* \to X^*$ such that $\|\tilde{T}\| = 1$, $\tilde{T}^*\hat{x}_i = \hat{x}_i$, $i = 1, \ldots, n$, and $\|\tilde{T}f_j - f_j\| < \varepsilon$, $j = 1, \ldots, m$.*

Proof. By subreflexivity there exist y_j , $j = 1, \ldots, m$, such that $\|f_j - f_{y_j}\| < \varepsilon$, $j = 1, \ldots, m$.

By Lemma 2 there exist a separable subspace C and a linear operator $T : C^* \to X^*$ such that $\|T\| = 1$, $\tilde{T}^*\hat{x}_i = \hat{x}_i$, $i = 1, \ldots, n$, $\tilde{T}^*\hat{y}_j = \hat{y}_j$, $j = 1, \ldots, m$. As $\tilde{T}^*\hat{y}_j = \hat{y}_j$, $j = 1, \ldots, m$, we have $f_{y_j} = \tilde{T}f_{y_j}$, $j = 1, \ldots, m$, so that $\|\tilde{T}f_j - f_j\| < \varepsilon$, $j = 1, \ldots, m$.

Before continuing we note an easy result.

LEMMA 4. *Let Y be a closed subspace of X . If $\overline{D_{X^*}(Y)}$ is a*

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linear subspace, then it is isometric to Y^* .

Proof. Let $T : \overline{D_{X^*}(Y)} \to Y^*$ be the restriction map. T is a linear norm preserving map of $\overline{D_{X^*}(Y)}$ into Y^* . That T is onto follows from the Hahn-Banach theorem as Y is subreflexive.

By the *density character* of a Banach space we mean the minimal cardinality of a dense subset.

LEMMA 5. *Let* X *be a smooth space and* M *be an infinite cardinal number. Suppose* Z, W *are subspaces of* X, X^* *respectively of density character not greater than* M . *Then there exists a subspace* C *of* X *of density character not greater than* M *which contains* Z , *together with a linear operator* $T : C^* \to X^*$ *such that* $P = \tilde{T}$ *is a bounded linear projection satisfying* $\|P\| = 1$, $Pf = f$ *for all* $f \in W$, $P^*\hat{x} = \hat{x}$ *for all* $x \in C$, *and such that* $PX^* = \overline{D_{X^*}(C)}$; *in particular,* PX^* *is isometric to the conjugate of* C .

Proof. The proof is by transfinite induction. Initially we assume that $\{f_j \ ; \ j = 1, 2, \ldots\}$ is dense in W , and that $\{x_j \ ; \ j = 1, 2, \ldots\}$ is dense in Z . By Lemma 3 we can construct inductively a sequence $\{C_n \ ; \ n = 1, 2, \ldots\}$ of separable subspaces of X and a sequence $\{T_n \ ; \ n = 1, 2, \ldots\}$ of linear operators $T_n : C_n^* \to X^*$, such that

(i) $\|T_n\| = 1$, $n = 1, 2, \ldots$,

(ii) $\tilde{T}_n^*\hat{x}_i = \hat{x}_i$, $1 \le i \le n$, $n = 1, 2, \ldots$,

$\tilde{T}_n^*\hat{x}_i^k = \hat{x}_i^k$, $1 \le i \le n$, $1 \le k \le n-1$, and

(iii) $\|\tilde{T}_n f_i - f_i\| < 1/n$, $1 \le i \le n$, $n = 1, 2, \ldots$,

where $\{x_i^k \ ; \ i = 1, 2, \ldots\}$ is dense in C_k , $k = 1, 2, \ldots$.

We let $C = \overline{sp}\left(\bigcup_{n=1}^{\infty} C_n\right)$ and we consider the extensions of $T_n, T_n' : C^* \to X^*$, $n = 1, 2, \ldots$, defined by $T_n'(f) = T_n(\underline{f})$, where $\underline{f}$

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is f restricted to C_n, $n = 1, 2, \ldots$. Following the technique of Lemma 2 we let T be a limit point in the $\hat{X}$-operator topology of the net $\{T'_n ; n = 1, 2, 3, \ldots\}$ and put $P = \tilde{T}$. It follows then that $\|P\| = 1$, P is linear and $P^*\hat{x}_i^k = \hat{x}_i^k$ for all i, k, so that $P^*\hat{x} = \hat{x}$ for all $x \in C$. As $P^*\hat{x} = \hat{x}$ for all $x \in C$ and $\|P\| = 1$ we obtain $Pf = f$ for all $f \in D_{X^*}(C)$ as X is smooth. As C is subreflexive it is now easily seen that $\overline{D_{X^*}(C)} = PX^*$ and that P is a projection. The last remark follows from Lemma 4.

We can assume now that the lemma holds for all cardinals less than M; we let Ω be the well-ordered set of ordinals less than M. There are closed subspaces $\{Z_\alpha ; \alpha \in \Omega\}$ of Z, $\{W_\alpha ; \alpha \in \Omega\}$ of W with $Z_\alpha \subset Z_\beta$, $W_\alpha \subset W_\beta$ for $\alpha < \beta$, such that the density characters of Z_α, W_α are at most the cardinality of α, for infinite α and such that $Z = \overline{\bigcup_{\alpha \in \Omega} Z_\alpha}$, $W = \overline{\bigcup_{\alpha \in \Omega} W_\alpha}$. By the induction hypothesis we can construct inductively for every $\alpha \in \Omega$ a subspace C_α of X whose density character is at most the cardinality of α for infinite α and such that $C_\alpha \supset Z_\alpha \cup \bigcup_{\beta<\alpha} C_\beta$, together with a linear operator

$$T_\alpha : C_\alpha^* \to X^*$$

such that $P_\alpha = \tilde{T}_\alpha$ satisfies the conditions $\|P_\alpha\| = 1$, $P_\alpha^*\hat{x} = \hat{x}$ for all $x \in C_\alpha$, $P_\alpha f = f$ for all $f \in W_\alpha$ and $P_\alpha X^* = \overline{D_{X^*}(C_\alpha)}$. We let $C = \overline{\bigcup_{\alpha \in \Omega} C_\alpha}$ and consider the extensions of T_α, $T'_\alpha : C^* \to X^*$ for each α. Again for T we take a limit in the $\hat{X}$-operator topology of the net $\{T'_\alpha ; \alpha \in \Omega\}$. T and C satisfy the conditions of the lemma.

Before proceeding we require two simple properties of Banach spaces with property A.

LEMMA 6. *Let Y be a Banach space with property A. Then the density character of Y^* is that of Y.*

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Proof. It is sufficient to check that the density character of Y^* is not greater than the density character M of Y. If Ω is the well-ordered set of ordinals less than M we may assume that $\{y_\alpha : \alpha \in \Omega\}$ is dense in Y. The set Φ consisting of all finite rational linear combinations of the elements f_{y_α} is a set of cardinality M. Furthermore Φ is dense in $D_{Y^*}(Y)$; for if $y \in Y$ there is a sequence $\left\{y_{\alpha_n} ; n = 1, 2, \ldots\right\}$ such that $y_{\alpha_n} \to y$ in norm, and hence, by property A, $f_{y_n} \to f_y$ in the weak topology, showing that f_y belongs to the closure of Φ by a result of Mazur [6, p. 422]. The lemma now follows as Y is subreflexive.

LEMMA 7. *Suppose* X *is a Banach space with property* A, *and that* $Y_\alpha \subset Y_\beta \subset X$ *for* $\alpha < \beta < \gamma$. *Then*

$$D_{X^*}\left(\overline{\bigcup_{\alpha<\gamma} Y_\alpha}\right) = \overline{\bigcup_{\alpha<\gamma} D_{X^*}(Y_\alpha)} ,$$

provided $\overline{\bigcup_{\alpha<\gamma} D_{X^*}(Y_\alpha)}$ *is a subspace.*

Proof. It suffices to show

$$D_{X^*}\left(\overline{\bigcup_{\alpha<\gamma} Y_\alpha}\right) \subset \overline{\bigcup_{\alpha<\gamma} D_{X^*}(Y_\alpha)} .$$

To establish this consider an element f_y where $y \in \overline{\bigcup_{\alpha<\gamma} Y_\alpha}$. Then there exists a sequence $\{y_n ; n = 1, 2, \ldots\} \subset \bigcup_{\alpha<\gamma} Y_\alpha$ such that

$$y_n \to y \text{ in norm.}$$

Property A ensures that $f_{y_n} \to f_y$ in the weak topology. The result follows as in Lemma 6 [6, p. 422].

We are now in a position to prove a theorem whereby it will be possible to reduce the proof of Theorem 1 to the separable case.

THEOREM 2. *Let* X *be a smooth space with property* A. *Let* μ *be*

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the first ordinal of cardinality the density character M *of* X . *For every* α *satisfying* $\omega \le \alpha < \mu$, *there is a subspace* X_α *of* X *of density character at most the cardinality of* α *together with a linear operator* $T_\alpha : X_\alpha^* \to X^*$ *such that* $P_\alpha = \tilde{T}_\alpha$ *is a bounded linear projection of* X^* *into* X^* *satisfying*

1. $\|P_\alpha\| = 1$,

2. $P_\alpha X^* = \overline{D_{X^*}(X_\alpha)}$, *and is thus isometric to* X_α^* ,

3. $P_\alpha P_\beta = P_\beta P_\alpha = P_\beta$ *where* $\beta < \alpha$,

4. $\bigcup_{\beta<\gamma} P_{\beta+1} X^*$ *is dense in* $P_\alpha X^*$, *for every* $\alpha > \omega$.

Moreover, $\bigcup_{\alpha<\mu} P_\alpha X^*$ *is dense in* X^* .

Proof. By Lemma 6 we may assume $\{f_\alpha \; ; \; \alpha < \mu\}$ is a dense subset of X^* . We construct $\{T_\alpha \; ; \; \omega \le \alpha < \mu\}$ by transfinite induction; if $M = \aleph_0$, $T_\omega = P_\omega = I$ has the required properties. Assume now that $M > \aleph_0$. By Lemma 5 there is a separable space X_ω together with a map T_ω such that $P_\omega = \tilde{T}_\omega$ satisfies $\|P_\omega\| = 1$, $P_\omega X^* = \overline{D_{X^*}(X_\omega)}$ and $P_\omega f_\alpha = f_\alpha$ for $\alpha < \omega$. Assume that $\{T_\beta \; ; \; \omega \le \beta < \gamma\}$ have been defined so that their extensions satisfy conditions 1 to 4.

If $\gamma = \alpha + 1$ we apply Lemma 5 to define X_γ and P_γ so that P_γ restricted to $P_\alpha X^* \cup \{f_\alpha\}$ is the identity and so that $X_\alpha \subset X_\gamma$. Lemma 5 is applicable by Lemma 6. It follows that $P_\gamma P_\beta = P_\gamma P_\alpha P_\beta = P_\alpha P_\beta = P_\beta$ for $\beta < \gamma$. Similarly $P_\gamma^* P_\beta^* = P_\beta^*$ follows as $\hat{X}_\alpha$ is ω^*-dense in X_α^{**} , [6, p. 425].

If on the other hand γ is a limiting ordinal, let $X_\gamma = \overline{\bigcup_{\alpha<\gamma} X_\alpha}$ and let $T'_\alpha : X_\gamma^* \to X^*$ be the extensions of T_α to X_γ^* for $\omega \le \alpha < \gamma$. For T_γ we then take a limit point in the $\hat{X}$-operator topology of the net

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$\{T'_\alpha ; \omega \in \alpha < \gamma\}$. Properties 1, 2 and 3 follow without difficulty whilst 4 holds by virtue of Lemma 7.

The last part now follows as $f_\alpha \in P_\omega X^*$ for $\alpha < \omega$ and $f_\alpha \in P_{\alpha+1}X^*$ for $\alpha \geq \omega$.

LEMMA 8. *Let* X *be a space with property* A *, and let* $\{P_\alpha ; \omega \leq \alpha < \mu\}$ *be the set of projections of* X^* *as in Theorem* 2. *Then for every* $f \in X^*$ *and every* $\varepsilon > 0$ *the set* $\{\alpha : \|P_{\alpha+1}f - P_\alpha f\| \geq \varepsilon\}$ *is finite.*

Proof. Assume, on the contrary, that there is an infinite sequence of ordinals $\omega \leq \alpha_1 < \alpha_2 < \ldots < \mu$ such that $\left\|P_{\alpha_{i+1}} f - P_{\alpha_i} f\right\| > \varepsilon$, $i = 1, 2, \ldots$. We denote P_{α_i} by P_{2i-1} , $P_{\alpha_{i+1}}$ by P_{2i} . Let

$X_\infty = \overline{\bigcup_{i=1}^{\infty} X_i}$ and consider the extensions of $T_i, T'_i : X^*_\infty \to X^*$, $i = 1, 2, \ldots$. If T_∞ is a limit point in the $\hat{X}$-operator topology of the sequence $\{T'_i ; i = 1, 2, \ldots\}$, then $P_\infty = \tilde{T}_\infty$ is a projection of X^* onto $\overline{\bigcup_{i=1}^{\infty} P_i X^*}$ and $P_i P_\infty = P_i$, $i = 1, 2, \ldots$.

If $h \in P_\infty X^*$, it follows that $\lim_i \|P_i h - h\| = 0$. For suppose that $g \in P_j X^*$ and that $\|g - h\| < \delta/2$. Then

$$\|P_i h - h\| \leq \|P_i h - P_i g\| + \|P_i g - g\| + \|g - h\| < \delta \quad \text{for} \quad i > j .$$

Hence $\lim\|P_i f - P_\infty f\| = \lim\|P_i P_\infty f - P_\infty f\| = 0$. But then $\{P_i f ; i = 1, 2, \ldots\}$ is a Cauchy sequence contradicting our assumption.

Proof of Theorem 1. The proof is by transfinite induction on the density character M of X or X^* . If $M = \aleph_0$ the result is well known; we may take $\{x_n ; n = 1, 2, \ldots\}$ to be a dense subsequence of the unit ball and put $(Tf)(n) = f(x_n)/n$.

We assume now that the theorem has been proved for all cardinals smaller than M . Let $\{P_\alpha ; \omega \le \alpha < \mu\}$ be the set of projections constructed in Theorem 2. As $P_\alpha X^*$ is isometric to X^*_α , the conjugate of a smooth space with property A , by the induction hypothesis there is a set Γ_α and a one-to-one linear operator T_α from $P_\alpha X^*$ into $c_0(\Gamma_\alpha)$. We may assume that the Γ_α are pairwise disjoint and that $\|T_\alpha\| \le 1$ for α satisfying $\omega \le \alpha < \mu$.

We put $\Gamma = N \cup \bigcup\{\Gamma_{\alpha+1} ; \omega \le \alpha < \mu\}$ and define $(Tf)(n) = (T_\omega P_\omega f)(n)$ for $n \in N$, and $(Tf)\gamma = 1/2\, T_{\alpha+1}(P_{\alpha+1}f - P_\alpha f)\gamma$ for $\gamma \in \Gamma_{\alpha+1}$. By Lemma 8, T maps X^* into $c_0(\Gamma)$, T is linear, and $\|T\| \le 1$. Furthermore if $Tf = 0$ then $P_\omega f = 0$ and $P_{\alpha+1}f = P_\alpha f$ for $\omega \le \alpha < \mu$. As $\bigcup_{\beta<\alpha} P_\beta X^*$ is dense in $P_\alpha X^*$ for every limiting $\alpha > \omega$, it follows by transfinite induction that $P_\alpha f = 0$ for all $\alpha < \mu$. But $\bigcup_{\alpha<\mu} P_\alpha X^*$ is dense in X^* so that $f = 0$. Hence T is one-to-one and the theorem is proved.

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