THE ROLE OF MAGNETIC FIELDS IN THE HEATING OF STELLAR ATMOSPHERES — THEORY

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ABSTRACT

The last ten years of observations have shown beyond doubt the fundamental role played by the magnetic field in the heating of stellar atmospheres. After the recognition of the extreme inhomogeneity of the solar corona, two basic new trends have appeared in the theoretical literature on the coronal heating problem. One is the adoption of a global point of view that stresses the connection of the properties of the upper layers to those of the underlying ones. In this way a general framework is provided, capable of accomodating many possible heating mechanisms that need not to be specified at this stage. The second novelty is the explicit inclusion in the theory of the inhomogeneous nature of the stellar envelopes, as a result of the presence of magnetic fields. The present status of knowledge on the subject as determined by the above evolution of the theoretical approach will be reviewed.

1. INTRODUCTION

The heating of the outer atmospheres of the Sun and the stars constitutes one of the outstanding unresolved problems of stellar physics. This problem has been on the astrophysical scene for such a long time without substantial progress that almost everybody working in the field felt discouraged one time or another. I believe that in the last few years this situation has changed: significant progress has been achieved and more is in sight. This review will try to substantiate the above optimistic statements.

Coronal heating, and in particular magnetic heating, has been the subject of a number of recent, exhaustive review papers (Heyvaerts and Schatzman 1980, Chiuderi 1981, Kuperus et al. 1981, Wentzel 1981, Priest 1982): we refer the reader to the above papers for a detailed presentatio of the general problem, a discussion of the relevant observations and an extensive bibliography. Here we shall simply outline the basic facts and concentrate mostly on the new aspects of the theoretical approach.

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The very existence of a stellar corona, namely of a hot plasma overlying a cooler one, indicates the need of a non-radiative heating mechanism. The identification of such a mechanism is the purpose of heating theories. The energetics does not pose any significant difficulty: given the very low heat capacity of a stellar corona almost any non-thermal process is capable of generating the required amount of energy. The difficult task is to describe correctly the spatial distribution of energy deposition and temperature.

The most important phenomenological constraints come from solar observations in the EUV and X-ray ranges. These have shown that the corona is a highly structured medium and that the structuring is intimately tied with the topology of the atmospheric magnetic fields. Most of the emission in the above mentioned wavelength ranges is concentrated in loop structures that form the magnetic active regions. These facts have forced the theoreticians to abandon the previously adopted homogeneous models and to investigate in detail the effects of the presence of the magnetic field both in its passive role of defining the different structures and in its active role of providing efficient heating mechanisms. There has been however a certain inertia in dropping the homogeneous assumption. Thus, the vast literature on coronal loops, generally treats them as homogeneous sub-systems without really investigating the actual consequences of the variation in space (and time) of the various physical quantities. Another shortcoming of many theoretical investigations on coronal loops is the fact that they are considered as entities separated from the underlying atmospheric layers. In spite of this, the study of coronal loops has produced a few important results and revealed the existence of simple relationships between the basic (and in principle measurable) physical parameters of the loop, such as length, pressure, temperature etc.

These relationships, or scaling laws, proved to be extremely useful to understand at least the gross features of solar coronal structures and especially in extending the knowledge thus gained to the stellar context. At the same time it become increasingly evident that no real progress toward the identification of the heating process, or processes, could be achieved without inclusion of the so far neglected aspects of the problem. The most important recent work on coronal heating, to be presented in the following, constitutes an effort to eliminate these unphysical limitations.

We shall first discuss the consequences of considering the coupling of the coronal plasma with the photospheric and sub-photospheric one. This provides a valuable general framework for the study of the heating problem. A number of important results can be obtained without specifying the actual physical mechanism at work. We shall then turn to the description of one particular mechanism, the phase-mixing of shear Alfvèn waves, that has been recently worked out. This calculation represents the quantitative counterpart of the more qualitative global description discussed in the first part. As we shall see the results fit perfectly in the general framework thus confirming its va-

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lidity and strengthening our confidence in its use in other astrophysical situations.

2. A GLOBAL DESCRIPTION

It has already been mentioned that the magnetic loops are the regions where most of the coronal emission is concentrated. It is however clear that the primary source of the radiated energy is not to be found within the loops themselves. Most, if not all, heating theories place the ultimate energy source in the photospheric or sub-photospheric regions, characterized by an intense mechanical activity in the form of material motions of a more or less ordered nature, from convection to turbulence. The two regions - corona and photosphere - are distinct by their difference in the value of the plasma parameter $\beta = (8\pi p/B^2) = 2(c_S/c_A)^2$ expressing the ratio between the thermal and magnetic energy densities. In the corona $\beta < 1$ whereas in the photosphere $\beta \gtrsim 1$. The magnetic field threads the whole stellar atmosphere and therefore electrodynamically couples the regions where mechanical activity dominates to those where heating is effective. The magnetic field thus provides a potentially powerful mean of transferring energy from material motions in the $\beta > 1$ regions into thermal energy in the $\beta < 1$ regions by driving various electrodynamical processes. Notice that in a mechanically coupled stellar atmosphere (acoustic wave heating) β should remain larger than unity over the entire envelope.

The adoption of a global point of view in which the behaviour of coronal structures is causally tied to the physics of the underlying layers is clearly the only one capable of providing a self-consistent solution of the heating problem. For a long time however it has been thought that the difficulties of such an approach were so overwhelming that a better way was to separate the source regions ($\beta > 1$) from the heated regions ($\beta < 1$) and to study the latter first by introducing an unknown heating function E_H to be determined from a comparison of theories and observations. This philosophy has produced an enormous amount of possible candidate heating mechanisms - such as Alfvén waves, fast and slow magnetosonic waves, surface waves, resistive instabilities etc. - each of which has its own merits and problems. A discrimination between them is extremely difficult from an observational point of view, the observables being subject to large errors or insensitive to the differences in the heating function. On the other hand no theoretical discrimination is possible without connecting in a coherent fashion the physics of the source regions to that of the heated regions.

A first important step in the direction of a global description has been recently achieved with the introduction of the concept of the equivalent electrodynamic circuit by J.A. Ionson (Ionson 1982 a, b). He has been able to show that the electrodynamic coupling between a coronal loop and its $\beta > 1$ "driver" can be represented phenomenologically in terms of a simple LRC driven circuit whose physics is well understood. The equivalent L,R and C can be expressed in terms of the basic

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parameters of the magnetic loop (the field-aligned scale length ℓ_z, the cross-field scale $\ell_\perp$, the magnetic field in the loop B_o). The driving emf on the other hand is expressed in terms of the $\beta > 1$ velocity field's spectral power function. It is then possible to deduce a phenomenological form of the heating functions, E_H. Consideration of the behaviour of the equivalent electrodynamic circuit shows that the global system acts as a high-quality resonator as we are now going to sketch.

The starting point are the standard dissipative MHD equations including viscosity as well as resistivity. By linearizing the equations and considering approximately shear disturbances $(\nabla \times E) \simeq 0$ it is possible to deduce an equation that describes the local electrodynamics of the system:

$$\frac{4\pi}{c^2} \frac{\partial^2 j_z}{\partial t^2} - \eta [1+H(1-\beta)\, Pr_m^\perp] \frac{\partial^3 j_z}{\partial r^2 \partial t} - H(1-\beta) \left(\frac{4\pi c_A^2}{c^2} \right) \frac{\partial^2 j_z}{\partial z^2} = H(\beta-1) \frac{\partial^3 (v_\theta B_{ph}/c)}{\partial z\, \partial r\, \partial t} \qquad (1)$$

Here the z-axis is in the direction of the ambient magnetic field, the perpendicular direction being characterized by r and x_θ. v_θ is the azimuthal component of the photospheric velocity field, the resistivity, $Pr_m^\perp = (4\pi\mu_\perp/\rho c^2 \eta)$ is the cross-field magnetic Prandtl number, B_{ph} the photospheric magnetic field and $H(x)$ the Heaviside step-function. c_A is the Alfvén speed.

The terms in Eq. (1) divide naturally in two types; those containing time derivatives of even order, similar to the inductive and capacitive reactances in an LRC circuit, and those containing time derivatives of odd order, analogous to the resistence and to an external source of emf that drives the currents.

Resonances occur when the inductive and capacitive reactance terms cancel each other, i.e. when

$$\frac{\partial}{\partial t} \simeq c_A \frac{\partial}{\partial z}$$

or

$$\omega \simeq \pi\, c_A/\ell_z \quad , \qquad (2)$$

ℓ_z being the field-aligned scale length. Since $c_A = c_A(r)$, the condition expressed by Eq. (2) can be satisfied only at special r-locations for each ω. It is important to notice that the condition expressed by Eq. (2) needs not to be necessarily satisfied. If this happen, however, for each given ω absorption of electromagnetic energy takes place only at special r locations within a spatial bandwidth Δr that can be estimated directly from Eq. (1) expanded about the spatial resonance. The result is

$$\Delta r/a = \pi^{1/3} \, [1+H(1-\beta) \, Pr_m^{\perp} \,]^{1/3} \, R_m^{-1/3} \quad ,$$

R_m being the magnetic Reynolds number defined as:

$$R_m = (4\pi c_A \, a^2/\eta c^2 \ell_z) \quad .$$

Here a is the spatial scale of the Alfvén speed:

$$a = \left[\frac{d(\ell n c_A)}{dr}\right]^{-1} \quad .$$

A description of the global electrodynamics is obtained by integrating Eq. (1) over the volume of the system. This integration is greatly simplified by the assumed occurrence of spatial resonances. The result of this integration can be expressed in the form of a global electrodynamic equation, given by

$$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = \frac{d\varepsilon}{dt} \tag{3}$$

where

$$I = \int_0^{\ell_z/2} 2\pi j_z(r) \, r \, d \, r$$

$$L = 4\ell_z/\pi c^2$$

$$R = \eta(1+Pr_m^{\perp})(\ell_z/\Delta r)^2$$

$$C = (\ell_z c^2/4\pi c_A^2)$$

$$\varepsilon\,(t) = \ell_\perp v_\theta B_{ph}/c$$

and $\ell_\perp$ is the photospheric cross-field spatial scale. Eq. (3) constitutes the basic equation for the discussion of the LRC circuit analog of a coronal loop. It predicts the possibility of global structure oscillations at a characteristic frequency

$$\omega_o = (LC)^{-1/2} = \pi c_A/\ell_z$$

The precise nature of these oscillations is not important in the present philosophy: one example are the Alfvén surface waves (Ionson, 1978, Wentzel, 1979). As in any resonant LRC circuit we can define a Q-value,

$$Q = (L/CR^2)^{1/2} \quad .$$

Energy is absorbed in a limited frequency bandwidth $\Delta\omega$ around the resonant frequency ω_o with

$$\frac{\Delta\omega}{\omega_o} = 1/Q \qquad (4)$$

A direct evaluation of Q shows that for typical coronal conditions $Q \gg 1$. Thus the loop-driver system behaves as a high-quality resonator.

It is also possible to evaluate the heating function E_H in terms of suitable time-averages of the driving emf, $\varepsilon(t)$. The result (Ionson, 1982) is

$$E_H = \left(\frac{2}{\pi \ell_\perp^2 \ell_z}\right) \frac{\langle \varepsilon^2 \rangle_{\omega_o} \omega_o}{(L/C)^{1/2}} \left(\frac{R}{R_{tot}}\right) \qquad (5)$$

where $R_{tot} = R + R_{ph}$ includes the effect of the photospheric resistance. An important feature is that the total power drain on the driver that equals $E_H(R_{tot}/R)$ is totally independent of dissipation. This is typical of all high-quality resonators, but should not be taken as an indication that the nature of the irreversibilities that determine R is not important. Their role is to fix the fraction $\Delta\omega/\omega_o$ of the available driving emf that interacts with the system in resonant conditions. The total power drain on the driver is however proportional to $\Delta\omega/R_{tot}$ which is independent of dissipation since $\Delta\omega \sim R_{tot}$. In other words, given a resonant condition, the system absorbs all the available energy within the allowed frequency bandwidth.

At this stage we have the basic missing ingredient in the phenomenological loop theories, i.e. E_H. The standard scaling laws (Rosner et al. 1978) can then be used to predict in a self-consistent manner the thermodynamic properties of a coronal loop. For instance, the following scaling laws are obtained for the maximum temperature, T , and base pressure, P :

$$T(k) = 2.10 \times 10^4 \left[T_{eff} \langle \frac{1}{2} \rho\, v_\theta^2 \rangle_{ph}\right]^{2/7}$$

$$P(\text{dyne cm}^{-2}) = (7.8 \times 10^3/\ell_z) \left[T_{eff} \langle \frac{1}{2} \rho\, v_\theta^2 \rangle_{ph}\right]^{6/7}$$

where T_{eff} is the effective blackbody temperature of the star and $\langle \frac{1}{2} \rho\, v_\theta^2 \rangle_{ph}$ is the average spectral power of the photospheric velocity field evaluated at the resonance. An application of the above scaling laws to the Sun produces a remarkably good agreement with the observations.

3. A WORKED-OUT EXAMPLE

Let us now turn from the previous phenomenological discussion to quantitative study of a particular heating mechanism that explicitly takes into account the fact that the corona is a structured medium. Alfvén waves present themselves as a very natural candidate for transpor-

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ting energy from the low-lying layers to the uppermost ones. The damping of Alfvén waves by frictional processes however is so small that their efficiency as a coronal heating mechanism has always been rather doubtful. The introduction of a structure perpendicular to the field lines, i.e. of a gradient of the Alfvén speed transverse to the field itself gives rise to a series of new and interesting phenomena, such as surface Alfvén waves (Wentzel 1979, Roberts 1981, Ionson 1978). Most of the past investigations, however, were concerned with the polarization which has a component of the displacement parallel to the direction of inhomogeneity, x. Quite recently (Heyvaerts and Priest 1982) a very interesting study has been performed on the propagation of shear Alfvén waves with a displacement perpendicular to x. The behaviour of these waves is amenable to a rather complete analytical investigation and leads to the possibility of a rapid damping due to phase-mixing as will be now explained.

Let us consider an equilibrium field $B_0(\mathbf{x})\, e_z$ and its response to motions in the y-directions at a fixed frequency ω. If the velocity is taken as $v(x,z,t)\, e_y$ the dissipative MHD (non-linear) equations give rise to the following simple propagation equation

$$\frac{\partial v}{\partial t^2} = c_A^2(\mathbf{x})\, \frac{\partial^2 v}{\partial z^2} + (\gamma_m + \gamma_v)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \frac{\partial v}{\partial t} \tag{6}$$

where γ_m and γ_v are the magnetic diffusivity and the kinematic viscosity, both assumed to be constant and small.

Without dissipation Eq. (6) implies that each magnetic surface =constant oscillates independently with a wave number

$$k_z(x) = \omega / c_A(x) \quad .$$

As the waves propagate along neighbouring surfaces at different x they become increasingly out of phase. Thus increasingly steeper x-gradients are generated up to the point where the transverse scale becomes so small that the dissipation mechanisms completely damp the wave. The basic physics is extremely simple and provides a beautiful example of the immense wealth of new possibilities that appear as soon as the unphysical assumption of perfect homogeneity is abandoned. The damping length, Λ, can be easily estimated in the long-wavelength, weak-damping and strong phase-mixing limit. The result for a purely viscous damping is $\Lambda = (6R)^{1/3}\, k_z^{-1}(x)$ where R is the Reynolds number. Numerically,

$$\Lambda(\mathrm{km}) \simeq 5\times10^3\; B^{5/3} \left(\frac{\nu_c}{\nu}\right)^{1/3} \left(\frac{\tau}{10}\right)^{2/3} \left(\frac{10^{10}}{n}\right)^{5/6} \left(\frac{2\times10^6}{T}\right)^{1/6} \left(\frac{9}{100}\right) \tag{7}$$

where B is expressed in gauss, ν_c and ν are the classical and actual viscosity coefficients, τ is the wave period in seconds, n the number density in cm^{-3}, T the temperature and <u>a</u> the transverse variation scale $(z\, dk_z/d_x)^{-1}$. Thus short-period waves (τ = 10 s) in weak fields damp efficiently over a distance of 5000 km. For longer periods and/or stronger fields this purely <u>laminar</u> mechanism does not seem to be an efficient one, even if one allows for an enhanced viscosity well above its classical value. Notice that the previous calculation refers to a field without boundaries in the z-direction and therefore does not mimic the situation in closed structures. In the latter case (coronal loops) standing waves will be formed due to the reflection from the transition regions near the footpoints of the loop. In this case then, each magnetic surface will oscillate with a frequency

$$\omega(\mathbf{x}) = k_z \, c_A(x)$$

with k_z fixed. Phase-mixing of these standing waves will now take place in time, rather than in space. An estimate of the laminar viscous damping time, τ_d , gives

$$\tau_d = (6R)^{1/3} \, \omega^{-1}(\mathbf{x}) \quad .$$

For the typical parameters used in the previous numerical example $\tau_D \simeq 20\,\tau$ which is short enough to be of interest. By completely solving the problem for the bounded case with a general excitation it is also possible to show that a state is produced in which the rate of dissipation exactly balances the rate at which the standing waves are produced, independently of the value of the dissipative coefficients. This very important result is in perfect agreement with the general phenomenological description, but now, we have gained a great deal in physical insight. The damping is produced by friction between close layers. Given sufficient time, phase mixing produces gradients as steep as necessary to match any dissipative efficiency, no matter how small.

Heyvaerts and Priest proceed further by observing that the damping terms depend strongly on the actual values of the dissipative coefficients. Thus an investigation is required to determine whether there are instabilities capable of creating small-scale structures in the Alfvén waves themselves that would enhance the values previously obtained for the damping lengths (or times) in the laminar case. Of particular importance are those instabilities that grow on a timescale shorter than the wave period. The results of a very elegant stability analysis for the standing waves case can be summarized as follows. Near a magnetic field node of a standing wave, where the velocity shear is maximum a Kelvin-Helmholtz instability is likely to develop, for typical coronal parameters. The induced turbulence effectively lowers the Reynolds number well below its laminar value so that damping times could be as short as a few periods. Analogously, tearing-mode instabilities are likely to develop at a velocity node, where the field shear is maximum, producing a similar shortening of the damping times.

4. CONCLUSIONS AND FUTURE PERSPECTIVES

We have tried to illustrate the basic reasons that make us believe that the coronal heating problem has possibly overcome a dead point. Looking backwards or reading past and recent reviews on the subject one realizes, as usual, that many of the ideas or schemes that now prove to be correct, or at least promising, were already there since a long time. Like in a puzzle, the pieces were there, but somehow we were unable to make them fit together.

In the last few years many people, including ourselves, pointed out that the corona was extremely inhomogeneous, but kept on making models of coronal loops that were piece-wise homogeneous with little consideration to what happened at the interfaces. Even in the cases where this class of phenomena was considered, the first examples studied were probably not the simplest or the most significant in connection with the heating problem. Similarly, the electric circuit analogy had already been used before. But those authors (see Alfvén 1977 and references therein) were mostly concerned with LR circuits and failed to recognize the importance of the capacitive properties of the coronal electrodynamic analog. Thus the possibility of exciting global oscillations was missed and the analogy of the behaviour of the coronal loops with that of a high-Q resonator was not apparent.

The framework offered by the LRC circuit analog shows that much can be gained by adopting a phenomenological description in which we do not pretend to treat the finer details and, so to speak, match the degree of theoretical finesse to what can be checked by observations. In fact the inclusion of phenomenological terms and functions in the basic equations is only meaningful if they can be directly determined by observations. In all other cases ambiguities are always present that undermine the reliability of the conclusions. Clearly, the global electrodynamical description is not the ultimate theory. It only links the corona and the photosphere. But there are still vital ingredients, the upwelling magnetic fields and the velocity of the turbulent motions, that are not introduced in a self-consistent way. To achieve this goal one should link the photosphere with the interior by producing convincing and reliable dynamo theories. We still not have them (see e.g. Gilman, 1981), but we can now wait for them because we can measure the missing quantities and use them for our theories on heating even if we do not understand exactly where they come from.

Another important piece of information is missing: the structure of coronal magnetic fields. In fact, all we know is the gross topology of coronal fields, via their influence on the thermodynamical state of the ambient plasma. Nothing is actually known on the fine structure. Since no direct measurement of coronal magnetic fields seems to be feasible in the near future, we must rely heavily on theoretical analyses. The deep connection between termodynamical and magnetic properties, whose importance can hardly be overestimated, has received only little attention so far (Einaudi et al. 1982).

Finally we would like to stress again the need of performing detailed, quantitative analyses of the type previously reviewed, along with more phenomenological studies. The comparison of the results gives strength to the simpler approach if they agree or indicates possible pitfalls or oversimplifications if they don't. In turn the more formal approach is much more effective if some general, more intuitive picture is present to provide a track.

In conclusion, something has changed in the long-standing game of understanding the heating of the stellar atmospheres. The last word has not been said, but there is a precise feeling that, after all, we are on the right track.

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DISCUSSION

SPRUIT: I have a comment on the equivalent circuit approach. In reducing the MHD equations to this extent one sweeps some things under the carpet. I am mostly worried about the neglect of a) stratification of the atmosphere, b) inhomogeneity of the field. Both have a very strong effect on wave propagation. They dominate the behaviour of the wave between the convection zone and the base of the corona. By averaging the equations over the volume one looses the ability to keep track of these effects.

CHIUDERI: In the equivalent circuit approach we certainly loose a certain number of effects. However, from the point of view of comparing theory with observations, such an approach is likely to be more adequate to the status of present day observational knowledge than a more detailed description.

SPICER: Nowhere do I see in these calculations the effects of cooling by expansion. In fact the $\mathbf{J}\cdot\mathbf{E}$ work term given in the talk does not include the $\mathbf{J}\cdot(\mathbf{v}\times\mathbf{B})$ work of expansion. I am wondering how this affects the calculations, since there is nothing published that demonstrates cooling by expansion.

IONSON: If you take a typical speed of expansion and calculate the expansion losses within the loop, you will find that they are small.

SPICER: The relevant question is whether the expansion time is shorter than the heating time. Observationally it is impossible to tell at this time whether a loop is stationary or the top of the loop is expanding while new field is expanding up replenishing the field that has already moved up. Also the best observations of loops, due to Skylab, do not have the time or spatial resolution to support the concept of a stationary non-expanding loop.

ROBERTS: I would like to comment upon the equivalent circuit concept, and to support the remarks made by Spruit with regard to the role of stratification ($g \neq 0$). As to stratification, this is of vital importance in governing the behaviour of waves in propagating from the photosphere to the corona. As to the equivalent circuit concept based upon linear theory, this is just another way of looking at an old problem: the nature of MHD waves in a coronal field. But the linearized MHD equations may be readily investigated for a magnetically-structured atmosphere, and for this approach there is no necessity to invoke the various approximations presented in the description of circuit theory. In fact, such a treatment for coronal magnetic inhomogeneities shows the presence of oscillations with phases of the order of the Alfvén speed (see the detailed treatment in Edwin and Roberts: 1982, *Solar Phys.* **76**, pp. 239-259) — the so-called fast body magnetoacoustic modes. The circuit analogy seems to lead to the same result in that it predicts an oscillation at about the right frequency, but the detailed form of the modes is not apparent. In general, I would like to see a close comparison of the results of circuit theory with the more usual (e.g. normal mode) analysis. That would be very interesting and instructive.

CHIUDERI: I agree with the comment that the circuit analogy is not a new "theory". It simply casts the problem in a simple way and provides a general framework that includes many possible mechanisms. It shows that a certain number of features are not the consequence of a specific heating mechanism but are common to all of them. These features then cannot be used to discriminate between various mechanisms. I think I already stressed the fact that quantitative analyses must accompany this type of more intuitive approach.

CAMPOS: The model presented is based on an LRC circuit. Dr. Spruit has stated that he liked better to speak in terms of Alfvén waves including stratification effects. I would like to remark that most of your results can be proved by considering Alfvén-gravity waves propagating vertically in an isothermal atmosphere, allowing for the exponential growth in Alfvén speed. The frequencies of standing modes are given by

$$\omega_n = \frac{c_A}{L} \frac{j_n}{2},$$

where j_n is a root of the Bessel function J_0. In your formula it appears that $\omega_0 = \pi c_A/L$, so that π is replaced by $j_n/2 = 1.2, 2.0$, etc. The waveforms are given by the Bessel function J_0, and give the high gradients, etc. (cf. *Solar Phys.*, 1983).

BENZ: There are nice coronal oscillations observed after flares in metric radio emission. This radiation is probably emitted by energetic electrons trapped in high coronal loops ($\gtrsim$ 100 000 km), which are modulated by global oscillations. The damping time (about 10 - 100 periods) fits nicely with your prediction. However, the period is observed to be 1 - 2 seconds, two orders of magnitude smaller than predicted.

CHIUDERI: At this stage of the theory it is very dangerous to make direct comparisons with observations. It is not obvious that the observations are actually related to global oscillations rather than to time variations in the distribution functions.