# THE EFFECT OF TOROIDAL MAGNETIC FIELDS IN THE OVERSHOOT LAYER ON THE EIGENFREQUENCIES OF STELLAR OSCILLATIONS

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ABSTRACT. The overshoot layer in stellar convection zones is slightly subadiabatic and can be considered as a stable region for storage of magnetic flux. Belvedere, Pidatella and Stix (1986) estimated the size of the overshoot layer and computed the magnetic field strength, beyond which toroidal flux tubes become unstable to buoyancy, for a number of main sequence spectral types ranging from F5 to K0. Here we estimate the relative frequency perturbation $\omega^M_{\ell m}/\omega_o$ of high order acoustic modes due to the presence of a non-oblique axisymmetric magnetic field in the overshoot layer. We find that $\omega^M_{\ell m}/\omega_o$ increases with the advancing spectral type, the predicted frequency splitting being large enough to be detected by observations, at least for the Sun.

We conclude that magnetic field induced frequency splitting of high order acoustic modes may well be due to a toroidal field of relatively moderate strength just beneath the bottom of the convection zone.

## 1. INTRODUCTION

As is well known, the overshoot layer in stellar convection zones is slightly subadiabatic and may be considered as a suitable region for storage of magnetic flux.

Pidatella and Stix (1986) applied a non-local formalism of the mixing length theory (Shaviv and Salpeter, 1973) to the lower part of the Sun's convection zone, described by the "fast model" of Belvedere et al. (1980), to compute the radius $r_o$ and the thickness D of the overshoot layer, and the treshold magnetic field strenght $B_o$ beyond which thin toroidal flux tubes become unstable to buoyancy according to a criterion derived by Spruit and Van Ballegooijen (1982).

Belvedere, Pidatella and Stix (1986) repeated these calculations for different main sequence spectral types ranging from F5 to K0.

Here we want to point out the effect that toroidal magnetic fields located in the overshoot layer of stellar convection zones should have on the eigenfrequencies of stellar oscillations.

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## 2. THE RELATIVE FREQUENCY PERTURBATION

We compute the relative frequency perturbation $\omega^M_{\ell m}/\omega_o$ of high order acoustic modes, according to Dziembowski and Goode (1984), in the presence of a non-oblique axisymmetric toroidal magnetic field $B_o$.

Equation (3.33) of Dziembowski and Goode (1984) is:

$$\frac{\omega^M_{\ell m}}{\omega_o} = a_{\ell m} < \frac{d}{d\ell nP} \left(\frac{P_M}{P}\right) > + b_{\ell m} < \left(0.1 - \frac{d\ \ell nr}{d\ \ell nP}\right) \frac{P_M}{P} > \qquad (1)$$

where $a_{\ell m}$ and $b_{\ell m}$ are coefficients, the general and asymptotic expressions of which are given by equations (3.34) and (3.35) of Dziembowski and Goode (1984) respectively, P is the gas pressure and $P_M$ the magnetic pressure.

In the thin layer approximation (overshoot layer thickness $D \ll R$ stellar radius) we get:

$$\frac{\omega^M_{\ell m}}{\omega_o} \simeq a_{\ell m} P_o \frac{\Delta(P_M/P)}{\Delta P} + b_{\ell m}\left(0.1 - \frac{P_o}{r_o} \frac{D}{\Delta P}\right) (P_M/P)_o \qquad (2)$$

where the symbol denotes $\Delta$ the variation of the physical quantities in the overshoot layer of thickness D and the suffix o stands for suitable mean values within the layer.

Since, according to the criterion by Spruit and Van Ballegooijen:

$$P_M/P = (B_o^2/2\ \mu P) \simeq \gamma\ |\nabla-\nabla_{ad}| \qquad (3)$$

we get:

$$\frac{\omega^M_{\ell m}}{\omega_o} \simeq a_{\ell m} \frac{P_o}{\Delta P} \gamma\ \Delta|\nabla-\nabla_{ad}| + b_{\ell m}\left(0.1 - \frac{P_o}{\Delta P} \frac{D}{r_o}\right)\gamma|\nabla-\nabla_{ad}|_o \qquad (4)$$

We computed $\omega^M_{\ell m}/\omega_o$ for a number of main sequence spectral types ranging from F5 to K0 and for several modes of different degree $\ell$ and azimuthal order m.

In all cases, the second term in equation (4) was negligible compared to the first, so that the relative frequency perturbation can be expressed, in a good approximation, as:

$$\frac{\omega^M_{\ell m}}{\omega_o} \simeq a_{\ell m} \frac{P_o}{\Delta P} \gamma\ \Delta|\nabla-\nabla_{ad}| \qquad (5)$$

Thus, in the presence of toroidal magnetic fields of marginal buoyancy instability strength $B_o$, located in the overshoot layer, the relative frequency perturbation depends essentially on the gradient of $|\nabla-\nabla_{ad}|$ along the layer.

## 3. RESULTS AND DISCUSSION

Table I shows the values of the physical quantities of interest computed following Belvedere, Pidatella and Stix (1986) for some main sequence spectral types. Lengths are in Mm and $B_o$ is in Tesla ($10^4$Gauss)

TABLE I

| Spectral type | R | $r_o$ | D | $P_o/\Delta P$ | $\Delta\lvert\nabla-\nabla_{ad}\rvert$ | $B_o$ |
|---|---|---|---|---|---|---|
| F5 | 837 | 825 | 2.0 | 1.9 | 0.10 | 0.8 |
| SUN | 696 | 523 | 12.5 | 3.3 | 0.10 | 5.0 |
| G5 | 650 | 448 | 12.5 | 3.2 | 0.10 | 9.0 |
| K | 592 | 280 | 12.1 | 4.9 | 0.09 | 46.9 |

Table II shows $\omega^M_{\ell m}/\omega_o$ for the same main sequence spectral types.

TABLE II

| Spectral |  |  | (large ℓ) |  |  |
|---|---|---|---|---|---|
| type | ℓ=1,m=1 | ℓ=5,m=1 | m/ℓ=½ | m/ℓ=1/5 | m/ℓ=1/10 |
| F5 | $1.4\ 10^{-2}$ | $1.1\ 10^{-3}$ | $6.9\ 10^{-3}$ | $1.2\ 10^{-3}$ | $3.1\ 10^{-4}$ |
| SUN | $2.4\ 10^{-2}$ | $1.8\ 10^{-3}$ | $1.2\ 10^{-2}$ | $2.1\ 10^{-3}$ | $5.2\ 10^{-4}$ |
| G5 | $2.3\ 10^{-2}$ | $1.8\ 10^{-3}$ | $1.1\ 10^{-2}$ | $2.0\ 10^{-3}$ | $5.0\ 10^{-4}$ |
| KO | $3.2\ 10^{-2}$ | $2.4\ 10^{-3}$ | $1.5\ 10^{-2}$ | $2.7\ 10^{-3}$ | $6.9\ 10^{-4}$ |

The main trends deduced from Table II are:

- $\omega^M_{\ell m}/\omega_o$ increases slightly with the advancing spectral type, this being a consequence of the increasing value of $B_o$.
- $\omega^M_{\ell m}/\omega_o$ decreases with decreasing m/ℓ, following the dependence of the geometrical factor $a_{\ell m}$.

For the Sun the value ℓ=50, corresponding to the depth of the bottom of the convection zone, has been adopted.

For the other spectral types, the values of ℓ corresponding to the bottom of the convection zones are expected to decrease with the advancing spectral type.

Note, however, that the numbers shown in Table II have been computed assuming a filling factor n≃1 in the overshoot layer (see Belvedere et al., 1986). If $n < 1$, the computed values should scale as n.

Unfortunately, at the present theory and observations cannot give us the value of n, but only the value of the product nf, f being the number of loops that each magnetic flux tube forms when erupting at the surface (Belvedere et al. 1986). Moreover, n could vary with the spectral type, this making things more complex. Further work is needed.

Still, the main conclusion one can draw from the present analysis is that magnetic field induced frequency splitting, large enough to be detected by observations (at least for the Sun), may be due to the presence of a toroidal field of moderate strength just beneath the bottom of the convection zone, in the overshoot layer, which, according to the current ideas, is a suitable site for accumulation of flux tubes.

This does not exclude, of course, the possibility of a large magnetic field inside the core.

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