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ARTICLE

Natural selection and innovation-driven growth

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Abstract

This paper examines how the interaction between natural selection, household education choices and R&D activities influences macroeconomic growth. We develop an innovation-driven growth model that integrates household heterogeneity in educational ability with endogenous fertility and the activation of innovation. Our findings reveal that households with lower educational abilities accumulate less human capital but have more offspring and initially gain a temporary evolutionary advantage. This demographic shift enhances the likelihood of innovation taking off; however, the resulting reduction in the share of high-ability households ultimately constrains R&D efforts and slows long-term economic growth. We empirically validate our theoretical model using cross-country data and instrumental variables, demonstrating that disparities in educational ability negatively impact education, innovation and growth over the long run. This study provides new insights into the complex dynamics between natural selection, endogenous fertility and economic development, with significant implications for both policy and theory.

Keywords: natural selection; education; innovation; endogenous takeoff

JEL classification: O30; O40

1. Introduction

Modern macroeconomic models often utilize a representative household or assume a fixed composition of heterogeneous households. However, when households with differing characteristics also have varying fertility rates, the composition of the population evolves over time. This process, referred to as natural selection, can have profound implications for the macroeconomy. This study investigates how household heterogeneity and natural selection influence economic growth, particularly through their effects on education, human capital accumulation and innovation.

Households differ in their attitudes toward education and their ability to transmit human capital across generations. These differences persist over time and shape economic outcomes. For instance, Alesina et al. (2021) show that family attitudes toward education can remain stable even after significant policy interventions. Unfortunately, not all households are equally endowed with the ability to accumulate human capital, leading to disparities in education and fertility choices. This raises important questions: how does household heterogeneity affect fertility decisions? And how do these decisions, in turn, impact technological progress and growth?

To address these questions, we develop an innovation-driven growth model that incorporates fertility choices, natural selection among heterogeneous households and endogenous activation of innovation. Our model extends the unified growth theory pioneered by Galor (2005, 2011, 2022), which posits that households differ in their ability to accumulate human capital. In our

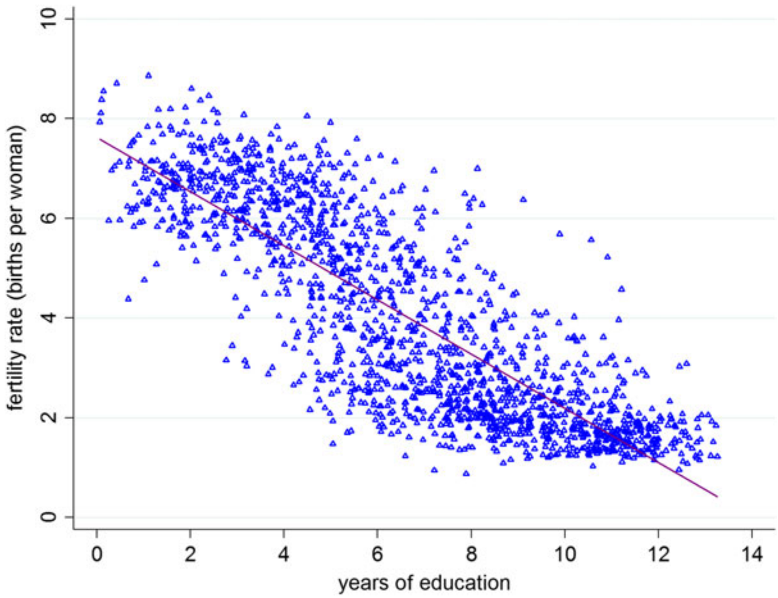

Figure 1. Fertility and education.
Notes: This figure depicts the negative correlation between fertility and education. The vertical axis represents the fertility rate, whereas the horizontal axis denotes the number of years of education.

framework, families with greater educational abilities prioritize child quality over quantity, resulting in fewer children but higher levels of human capital. Using panel data from 137 countries, we illustrate this well-documented negative relationship between fertility and education in Figure 1. This quality-quantity tradeoff, also supported by empirical evidence from studies such as Becker et al. (2010), Fernihough (2017) and Klemp and Weisdorf (2019), implies an evolutionary advantage to families who choose to have more children but less education. We use our growth-theoretic framework to explore how this evolutionary process affects human capital accumulation, innovation and economic growth.

In our model, the evolutionary disadvantage of households with higher educational abilities are temporary. As the economy progresses, they eventually accumulate more human capital, leading to a convergence in fertility rates across households and a stationary distribution of population shares. One of the key insights of our model is that household heterogeneity can initially enhance the likelihood of innovation by increasing the total amount of human capital available for production and research and development (R&D) activities. However, this advantage is offset by the evolutionary disadvantage faced by high-ability households during transitional dynamics, which leads to a decline in their population share and, consequently, a reduction in the overall level of human capital in the economy. This evolutionary process results in a population that is, on average, less educated than it would be in the absence of natural selection, with significant implications for long-term economic growth. This finding resonates with the following observation: "Britons are becoming less educated and poorer because smart rich people are having fewer children."[1] A contribution of this study is to show that this phenomenon may be universal and not be specific to Britain.

Our findings also suggest that the temporary evolutionary advantage enjoyed by lower-ability households due to higher fertility rates has permanent effects on the economy's growth trajectory. Specifically, the scale-invariant nature of our model implies that the economy's steady-state growth rate is lower when the share of high-ability households decreases over time. We provide empirical evidence supporting this theory, demonstrating that heterogeneity in educational abilities adversely affects education, innovation, and economic growth in the long run. These results

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hold even when ancestral population diversity and prehistoric migratory distances in Ashraf and Galor (2013) are used as instrumental variables for educational heterogeneity.

Furthermore, this study not only advances the theoretical understanding of the dynamics between natural selection, household heterogeneity and economic growth but also provides actionable insights for addressing real-world challenges. Specifically, our findings suggest that interventions aimed at reducing educational disparities—such as meritocratic reforms and targeted public investments—can mitigate the evolutionary disadvantage faced by high-ability households and foster sustained innovation and growth.

This study contributes to the literature on innovation and economic growth by examining the complex interplay between natural selection, household heterogeneity, and R&D activities. The pioneering studies in this literature are Romer (1990), Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom et al. (1990) . We follow subsequent studies, such as Jones (2001), Connolly and Peretto (2003), Chu et al. (2013), Peretto and Valente (2015) and Brunnschweiler et al. (2021), by introducing endogenous fertility to the innovation-driven growth model in order to explore how endogenous fertility decisions among heterogeneous households shape economic development. A novelty of our analysis is that we allow for heterogeneous households, which give rise to an evolutionary process. We find that the temporary disadvantage faced by high-ability households during transitional periods has lasting consequences for technological progress and long-term growth. Our model provides new insights into these dynamics, offering valuable implications for both economic policy and the theoretical understanding of growth mechanisms.

Our work also relates to the broader literature on endogenous growth and economic transitions. An early study by Galor and Weil (2000) develops the unified growth theory that explores the endogenous transition of an economy from pre-industrial stagnation to modern economic growth; see Galor (2005, 2011, 2022) for a comprehensive review of unified growth theory and also Galor and Moav (2001, 2002), Galor and Mountford (2008), Galor, et al. (2009), Ashraf and Galor (2011), Galor and Michalopoulos (2012) and Carillo et al. (2019) for subsequent studies and empirical evidence that supports unified growth theory. For example, Galor and Moav (2002), Galor and Michalopoulos (2012) and Carillo et al. (2019) explore how natural selection of different traits, such as the quality preference of fertility, the degree of risk aversion and the level of family-specific human capital, affects the transition from stagnation to growth. This study complements these interesting studies by examining how natural selection of heterogeneous households with different ability to accumulate human capital affects the transition of an economy from human capital accumulation to modern economic growth that is driven by R&D and innovation.

Therefore, we also contribute to the related branch of the literature on the endogenous transition from pre-industrial stagnation to modern innovation-driven economic growth. For example, Funke and Strulik (2000) and Peretto (2015) explore how economies transition through different stages of development, including capital accumulation and innovation.[2] Our study adds to this literature by introducing natural selection to a tractable innovation-driven growth model, highlighting the role of household heterogeneity in shaping these transitions.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 discusses the two stages of economic development. Section 4 examines the implications of household heterogeneity and natural selection. Section 5 provides empirical evidence supporting our theoretical findings. Finally, Section 6 concludes.

2. An R&D-based growth model with natural selection

To model natural selection, we introduce heterogeneous households and endogenous fertility to the seminal Romer model. To keep the model tractable, we consider a simple structure of overlapping generations (OLG) and human capital accumulation.[3] Each individual lives for three periods. In the young age, the individual accumulates human capital. In the working age, the individual allocates her time between work, fertility and education of the next generation. In the old age,

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the individual consumes her saving. Saving is required in the OLG model of innovation-driven growth because inventions are owned by agents as assets.

2.1 Heterogeneous households

There is a unit continuum of households indexed by $i \in [0, 1]$. Within household i, the utility of an individual who works at time t is given by[4]

$$U^t(i) = u\left[n_t(i), h_{t+1}(i), c_{t+1}(i)\right] = \eta \ln n_t(i) + \gamma \ln h_{t+1}(i) + \ln c_{t+1}(i), \tag{1}$$

where $c_{t+1}(i)$ is the individual's consumption at time $t+1$, $n_t(i)$ denotes the number of children the individual has at time t, $\eta > 0$ is the fertility preference parameter, $h_{t+1}(i)$ denotes the level of human capital that the individual passes onto each child, and γ is the quality preference parameter. We assume that all individuals within the same household i have the same level of human capital at time 0. Then, they will also have the same level of human capital for all t as an endogenous outcome.

The individual allocates $e_t(i)$ units of time to her children's education. The accumulation equation of human capital is given by[5]

$$h_{t+1}(i) = \phi(i)e_t(i) + (1-\delta)h_t(i), \tag{2}$$

where the parameter $\delta \in (0, 1)$ is the depreciation rate of human capital that a generation passes onto the next.[6,7] As for the ability parameter $\phi(i) > 0$ of household i,[8] it is heterogeneous across households and follows a general distribution with the following mean:[9]

$$\overline{\phi} \equiv \int_0^1 \phi(i)di.$$

The heterogeneity of households is captured by their differences in $\phi(i)$, which in turn give rise to an endogenous distribution of human capital. We focus on heterogeneity in $\phi(i)$ because it allows for a stationary distribution of the population share of different households in the long run, whereas heterogeneity in other parameters, such as η or γ, imply that households with the largest η or smallest γ would dominate the population in the long run.

An individual in household i allocates $1 - e_t(i) - \sigma n_t(i)$ units of time to work and earns $w_t\left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i)$ as real wage income, where the parameter $\sigma \in (0, 1)$ determines the time cost $\sigma n_t(i)$ of fertility.[10] For simplicity, we assume that there are economies of scale in the time spent in educating children within a family, and the cost of having more children is reflected in the time cost of childrearing.[11]

The individual devotes her entire wage income to saving at time t and consumes the return at time $t+1$:[12]

$$c_{t+1}(i) = (1 + r_{t+1})w_t\left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i), \tag{3}$$

where r_{t+1} is the real interest rate. Substituting (2) and (3) into (1), the individual maximizes

$$\max_{e_t(i),\, n_t(i)} U^t(i) = \eta \ln n_t(i) + \gamma \ln\left[\phi(i)e_t(i) + (1-\delta)h_t(i)\right] + \ln\left\{(1 + r_{t+1})w_t\left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i)\right\},$$

taking $\{r_{t+1}, w_t, h_t(i)\}$ as given. The utility-maximizing level of fertility $n_t(i)$ is

$$n_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right], \tag{4}$$

which is decreasing in $\phi(i)$ but increasing in $h_t(i)$. In other words, households with a lower ability to accumulate human capital and a higher level of human capital choose to have more children. In (4), fertility $n_t(i)$ is decreasing in $\phi(i)/h_t(i)$. As we will show, households with higher $\phi(i)$ have

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higher $h_t(i)$ and also higher $\phi(i)/h_t(i)$ before the level of human capital reaches the steady state, at which point all households share the same $\phi(i)/h_t(i)$. Therefore, households with higher ability $\phi(i)$ generally have higher human capital $h_t(i)$ and lower fertility $n_t(i)$, generating a negative relationship between these two variables. To understand this negative relationship, we also derive the utility-maximizing level of education $e_t(i)$ as[13]

$$e_t(i) = \frac{1}{1+\eta+\gamma}\left[\gamma - (1+\eta)(1-\delta)\frac{h_t(i)}{\phi(i)}\right], \tag{5}$$

which is increasing in $\phi(i)$ but decreasing in $h_t(i)$. In summary, for a given $h_t(i)$, households with a larger $\phi(i)$ choose a higher level of education $e_t(i)$ but a smaller number $n_t(i)$ of children, reflecting the quality-quantity tradeoff. Given the same initial human capital $h_0(i) = h_0$, differences in education ability $\phi(i)$ give rise to differences in education level $e_t(i)$.[14]

Substituting (5) into (2) yields the autonomous and stable dynamics of human capital as

$$h_{t+1}(i) = \frac{\gamma}{1+\eta+\gamma}\left[\phi(i) + (1-\delta)h_t(i)\right], \tag{6}$$

where $h_{t+1}(i)$ is increasing in $\phi(i)$ and $h_t(i)$. The total amount of human capital in the economy at time t is

$$H_t = \int_0^1 h_t(i)L_t(i)di,$$

where $L_t(i)$ is the working-age population size of household i. The law of motion for $L_t(i)$ is

$$L_{t+1}(i) = n_t(i)L_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right]L_t(i), \tag{7}$$

and the size of the aggregate labor force in the economy at time t is

$$L_t = \int_0^1 L_t(i)di.$$

Let's define $s_t(i) \equiv L_t(i)/L_t$ as the working-age-population (i.e., labor) share of household i.

Lemma 1. *The labor share $s_t(i)$ of household i at time $t \geq 1$ is given by*

$$s_t(i) = \frac{\prod_{\tau=0}^{t-1} n_\tau(i)L_0(i)}{\int_0^1 \prod_{\tau=0}^{t-1} n_\tau(i)L_0(i)di},$$

where the fertility decision $n_t(i)$ of household i at time $t \geq 1$ is given by

$$n_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{\sum_{\tau=0}^{t-1}\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^\tau + \left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^t\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\},$$

which is a decreasing function of $\phi(i)/h_0(i)$.

Proof. *See Appendix A.* □

Notice that changes to $n_\tau(i)$ in any one period will affect $s_t(i)$ in all future generations. The reason is general and does not depend on the specific assumptions of this model: a temporary growth effect has a permanent level effect. Therefore, if the fertility rate of an ability group drops temporarily, this group would *ceteris paribus* forever have a lower population share than it would otherwise have had. As we will later see, if the high-ability household experiences a temporary reproduction loss, the economy will have a lower share of high-ability people forever. We will also show that this loss will permanently lower human capital, innovation and growth.

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2.2 Final good

Perfectly competitive firms use the following production function to produce final good Y_t, which is chosen as the numeraire:

$$Y_t = H_{Y,t}^{1-\alpha} \int_0^{N_t} X_t^{\alpha}(j)dj, \tag{8}$$

where the parameter $\alpha \in (0, 1)$ determines production labor intensity $1-\alpha$, and $H_{Y,t}$ denotes human-capital-embodied production labor. $X_t(j)$ denotes a continuum of differentiated intermediate goods indexed by $j \in [0, N_t]$. Firms maximize profit, and the conditional demand functions for $H_{Y,t}$ and $X_t(j)$ are given by

$$w_t = (1-\alpha)\frac{Y_t}{H_{Y,t}}, \tag{9}$$

$$p_t(j) = \alpha \left[\frac{H_{Y,t}}{X_t(j)}\right]^{1-\alpha}. \tag{10}$$

2.3 Intermediate goods

Each intermediate good j is produced by a monopolistic firm, which uses a one-to-one linear production function that transforms $X_t(j)$ units of final good into $X_t(j)$ units of intermediate good $j \in [0, N_t]$. The profit function is

$$\pi_t(j) = p_t(j)X_t(j) - X_t(j), \tag{11}$$

where the marginal cost of production is constant and equal to one (recall that final good is the numeraire). The monopolist maximizes (11) subject to (10) to derive the monopolistic price as

$$p_t(j) = \min\left\{\mu, \frac{1}{\alpha}\right\} = \mu > 1, \tag{12}$$

where $\mu \leq 1/\alpha$ is a patent policy parameter as in Li (2001) and Goh and Olivier (2002). One can show that $X_t(j) = X_t$ for all $j \in [0, N_t]$ by substituting (12) into (10). Then, we substitute (10) and (12) into (11) to derive the equilibrium amount of monopolistic profit as

$$\pi_t = (\mu - 1)\, X_t = (\mu - 1)\left(\frac{\alpha}{\mu}\right)^{1/(1-\alpha)} H_{Y,t}. \tag{13}$$

2.4 R&D

We denote v_t as the value of a newly invented intermediate good at the end of time t. The value of v_t is given by the present value of future profits from time $t+1$ onwards:

$$v_t = \sum_{s=t+1}^{\infty}\left[\pi_s / \prod_{\tau=t+1}^{s}(1+r_\tau)\right]. \tag{14}$$

Competitive R&D entrepreneurs invent new products by employing $H_{R,t}$ units of human-capital-embodied labor. We specify the following innovation process:[15]

$$\Delta N_t = \frac{\theta N_t H_{R,t}}{L_t}, \tag{15}$$

where $\Delta N_t \equiv N_{t+1} - N_t$. The parameter $\theta > 0$ determines R&D productivity $\theta N_t / L_t$, where N_t captures intertemporal knowledge spillovers as in Romer (1990) and $1/L_t$ captures a dilution effect

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that removes the scale effect.[16] If the following free-entry condition holds:

$$\Delta N_t v_t = w_t H_{R,t} \Leftrightarrow \frac{\theta N_t v_t}{L_t} = w_t, \tag{16}$$

then R&D $H_{R,t}$ would be positive at time t. If $\theta N_t v_t / L_t < w_t$, then R&D does not take place at time t (i.e., $H_{R,t} = 0$). Lemma 2 provides the condition for $H_{R,t} > 0$, which requires R&D productivity θ to be sufficiently high in order for innovation to take place.

Lemma 2. *R&D $H_{R,t}$ is positive at time t if and only if the following inequality holds:*

$$\int_0^1 [1 - e_t(i) - \sigma n_t(i)]\, h_t(i) s_t(i) di > \frac{1}{\theta}. \tag{17}$$

Proof. *See Appendix A.* □

2.5 Aggregation

Imposing symmetry on (8) yields $Y_t = H_{Y,t}^{1-\alpha} N_t X_t^{\alpha}$. Then, we substitute (10) and (12) into this equation to derive the aggregate production function as

$$Y_t = \left(\frac{\alpha}{\mu}\right)^{\alpha/(1-\alpha)} N_t H_{Y,t}. \tag{18}$$

Using $N_t X_t = (\alpha/\mu) Y_t$, we obtain the following resource constraint on final good:

$$C_t = Y_t - N_t X_t = \left(1 - \frac{\alpha}{\mu}\right) Y_t, \tag{19}$$

where C_t denotes aggregate consumption. Finally, the resource constraint on human-capital-embodied labor is

$$\int_0^1 [1 - e_t(i) - \sigma n_t(i)]\, h_t(i) L_t(i) di = H_{Y,t} + H_{R,t}. \tag{20}$$

2.6 Equilibrium

The equilibrium is a sequence of allocations $\{X_t(j), Y_t, e_t(i), n_t(i), c_t(i), C_t, h_t(i), H_t, H_{Y,t}, H_{R,t}, L_t\}$ and prices $\{p_t(j), w_t, r_t, v_t\}$ that satisfy the following conditions:

- individuals choose $\{e_t(i), n_t(i), c_t(i)\}$ to maximize utility taking $\{r_{t+1}, w_t, h_t(i)\}$ as given;
- competitive firms produce Y_t to maximize profit taking $\{p_t(j), w_t\}$ as given;
- a monopolistic firm produces $X_t(j)$ and chooses $p_t(j)$ to maximize profit;
- competitive entrepreneurs perform R&D to maximize profit taking $\{w_t, v_t\}$ as given;
- the market-clearing condition for the final good holds such that $Y_t = N_t X_t + C_t$;
- the resource constraint on human-capital-embodied labor holds such that $H_{Y,t} + H_{R,t} = \int_0^1 [1 - e_t(i) - \sigma n_t(i)]\, h_t(i) L_t(i) di$;
- total saving equals asset value such that $w_t \int_0^1 [1 - e_t(i) - \sigma n_t(i)]\, h_t(i) L_t(i) di = N_{t+1} v_t$.

3. Stages of economic development

Our model features two stages of economic development. The first stage features only human capital accumulation. The second stage features both human capital accumulation and innovation.[17]

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The activation of innovation and the resulting transition from the first stage to the second stage are endogenous and do not always occur.

3.1 Stage 1: Human capital accumulation only

The initial level of human capital for each individual in household i is $h_0(i)$. Suppose the following inequality holds at time 0:

$$\int_0^1 [1 - e_0(i) - \sigma n_0(i)]\, h_0(i) s_0(i) di = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_0(i)}{\phi(i)}\right] h_0(i) s_0(i) di < \frac{1}{\theta}, \quad (21)$$

which uses (4) and (5). In (21), both the initial labor share $s_0(i) \equiv L_0(i)/L_0$ and initial human capital $h_0(i)$ are exogenously given. Then, Lemma 2 implies that $H_{R,0} = 0$ and

$$H_{Y,0} = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_0(i)}{\phi(i)}\right] h_0(i) L_0(i) di. \quad (22)$$

In this stage of development, the economy features only human capital accumulation. Human capital $h_t(i)$ accumulates according to the autonomous and stable dynamics in (6), and $s_t(i)$ evolves according to Lemma 1. However, so long as the following inequality holds at time t:

$$\int_0^1 [1 - e_t(i) - \sigma n_t(i)]\, h_t(i) s_t(i) di = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) s_t(i) di < \frac{1}{\theta}, \quad (23)$$

we continue to have $H_{R,t} = 0$ and

$$H_{Y,t} = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) L_t(i) di. \quad (24)$$

Substituting (24) into (18) yields the level of output per worker as

$$y_t \equiv \frac{Y_t}{L_t} = \left(\frac{\alpha}{\mu}\right)^{\alpha/(1-\alpha)} N_0 \frac{H_{Y,t}}{L_t} = \left(\frac{\alpha}{\mu}\right)^{\alpha/(1-\alpha)} \frac{N_0}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) s_t(i) di, \quad (25)$$

where N_0 remains at the initial level and output increases as human capital accumulates.

3.2 Does innovation occur?

Equation (6) shows that human capital $h_t(i)$ converges to a steady state given by

$$h^*(i) = \frac{\gamma \phi(i)}{1+\eta+\gamma\delta}, \quad (26)$$

which is increasing in household i's ability $\phi(i)$. Substituting (26) into (4) and (5) yields the steady-state levels of education and fertility given by

$$e^*(i) = e^* = \frac{\gamma\delta}{1+\eta+\gamma\delta}, \quad (27)$$

$$n^*(i) = n^* = \frac{\eta}{\sigma(1+\eta+\gamma\delta)}, \quad (28)$$

where n^* is the same across all households because they are independent of $\phi(i)$. In other words, the negative effect of $\phi(i)$ and the positive effect of $h^*(i)$ on $n^*(i)$ cancel each other. As a result, the distribution of the population share of different households is stationary in the long run.

In the long run, we may have positive or negative population growth. If we assume $\eta > (1+\gamma\delta)\sigma/(1-\sigma)$, then the long-run population growth rate would be positive (i.e., $n^* > 1$).

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If we assume $\eta < (1+\gamma\delta)\sigma/(1-\sigma)$ instead, then the long-run population growth rate would be negative (i.e., $n^* < 1$). Even with negative population growth in the long run, the economy may still experience economic growth (i.e., long-run growth in y_t) driven by innovation.[18]

Does innovation occur? It depends on R&D productivity θ. Lemma 2 implies that if the following inequality holds:

$$\left(1 - e^* - \sigma n^*\right) \int_0^1 h^*(i)s^*(i)di = \frac{\gamma}{(1+\eta+\gamma\delta)^2} \int_0^1 \phi(i)s^*(i)di > \frac{1}{\theta}, \tag{29}$$

then human capital accumulation eventually triggers the activation of innovation, under which the R&D condition in (16) holds and R&D $H_{R,t}$ becomes positive. Therefore, the endogenous activation of innovation requires a sufficiently large R&D productivity parameter θ, such that (29) holds before human capital converges to a steady state. If innovation does not occur, then the economy features only human capital accumulation and converges to the following steady-state level of output per worker:

$$y^* = \left(\frac{\alpha}{\mu}\right)^{\alpha/(1-\alpha)} \frac{\gamma N_0}{(1+\eta+\gamma\delta)^2} \int_0^1 \phi(i)s^*(i)di,$$

which uses (26) in (25).

3.3 Stage 2: Innovation and human capital accumulation

We now consider the case in which the activation of innovation has occurred and derive the equilibrium growth rate in the presence of innovation. Substituting (18) into (9) yields the equilibrium wage rate as

$$w_t = (1-\alpha)\left(\frac{\alpha}{\mu}\right)^{\alpha/(1-\alpha)} N_t. \tag{30}$$

Then, substituting (30) into (16) yields the equilibrium invention value as

$$\frac{v_t}{L_t} = \frac{1-\alpha}{\theta}\left(\frac{\alpha}{\mu}\right)^{\alpha/(1-\alpha)}. \tag{31}$$

The structure of overlapping generations implies that the value of assets at the end of time t must equal the amount of saving at time t given by wage income at time t:

$$N_{t+1}v_t = w_t \int_0^1 \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i)L_t(i)di = w_t(H_{Y,t} + H_{R,t}), \tag{32}$$

where the second equality uses (20). Substituting (30) and (31) into (32) yields

$$N_{t+1} = \frac{\theta N_t}{L_t}(H_{Y,t} + H_{R,t}). \tag{33}$$

Combining (15) and (33) yields the equilibrium level of $H_{Y,t}$ as

$$\frac{H_{Y,t}}{L_t} = \frac{1}{\theta} \tag{34}$$

for all t. Substituting (4), (5) and (34) into (20) yields the equilibrium level of $H_{R,t}$ as

$$\begin{aligned} \frac{H_{R,t}}{L_t} &= \int_0^1 \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i)s_t(i)di - \frac{H_{Y,t}}{L_t} \\ &= \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i)s_t(i)di - \frac{1}{\theta}. \end{aligned} \tag{35}$$

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We can now substitute (35) into (15) to derive the equilibrium growth rate of N_t as

$$g_t \equiv \frac{\Delta N_t}{N_t} = \frac{\theta H_{R,t}}{L_t} = \frac{\theta}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) s_t(i) di - 1, \tag{36}$$

which is also the equilibrium growth rate of output per worker $y_t = (\alpha/\mu)^{\alpha/(1-\alpha)} N_t/\theta$. Finally, the steady-state equilibrium growth rate of N_t and y_t is

$$g^* = \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2} \int_0^1 \phi(i) s^*(i) di - 1. \tag{37}$$

In the steady state, $s^*(i)$ is also the population share of household i and still depends on the initial distribution of $h_0(i)$ and the exogenous distribution of $\phi(i)$ as shown in Lemma 1.

4. Heterogeneous households and evolutionary differences

Equation (21) shows that the activation of innovation-driven growth occurs at time 0 if and only if the following inequality holds:

$$\frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_0(i)}{\phi(i)}\right] h_0(i) s_0(i) di > \frac{1}{\theta}. \tag{38}$$

Suppose we consider a useful benchmark of an equal initial labor share $s_0(i) = 1$ and an equal initial level of human capital $h_0(i) = h_0$ for all $i \in [0, 1]$. Then, the left-hand side of (38) simplifies to

$$\frac{h_0}{1+\eta+\gamma} \left[1 + (1-\delta) h_0 \int_0^1 \frac{1}{\phi(i)} di\right] > \frac{h_0}{1+\eta+\gamma} \left[1 + \frac{(1-\delta)h_0}{\overline{\phi}}\right], \tag{39}$$

where $\int_0^1 [1/\phi(i)] di > 1/\overline{\phi}$ due to Jensen's inequality. In other words, the presence of heterogeneity in $\phi(i)$ makes the activation of innovation-driven growth more likely to occur at time 0 than the absence of heterogeneity (i.e., $\phi(i) = \overline{\phi}$ for all $i \in [0, 1]$) does. Due to heterogeneity, some households supply more human capital for production and innovation while others supply less. Equation (39) implies that the former effect dominates the latter effect such that the initial amount of human capital available for production and innovation increases as a result of heterogeneity. The intuition can be explained as follows.

Although some low-ability households may devote almost no time to education and most of their time to work (and fertility), high-ability households always spend some time to work, as the following shows:

$$1 - e_0(i) - \sigma n_0(i) = \frac{1}{1+\eta+\gamma} \left[1 + \frac{(1-\delta)h_0}{\phi(i)}\right] > \frac{1}{1+\eta+\gamma} > 0.$$

The convexity of $1/\phi(i)$ in $1 - e_0(i) - \sigma n_0(i)$ gives rise to the positive effect of heterogeneity on the amount of human capital available for production and innovation. To put it differently, the low-ability households being less willing to educate their children contribute to a larger workforce, which in turn rewards the innovation pioneers with more profits extracted from a larger market size of the economy. We summarize this result in the following lemma.

Lemma 3. *Heterogeneity makes it more likely for innovation to be activated at time 0.*

Proof. *If the following inequality holds:*

$$\frac{h_0}{1+\eta+\gamma} \left[1 + (1-\delta) h_0 \int_0^1 \frac{1}{\phi(i)} di\right] > \frac{1}{\theta} > \frac{h_0}{1+\eta+\gamma} \left[1 + \frac{(1-\delta)h_0}{\overline{\phi}}\right], \tag{40}$$

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which is a nonempty parameter space due to $\int_0^1 [1/\phi(i)]di > 1/\overline{\phi}$, then the takeoff of the economy occurs at time 0 under heterogeneous households but not under homogeneous households. □

Next we examine how the labor share of households evolves over time. Given the benchmark of an equal initial labor share $s_0(i) = 1$ and an equal initial level of human capital $h_0(i) = h_0$ for all $i \in [0, 1]$, the fertility of household i at time 0 is

$$n_0(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left[1 + (1-\delta)\frac{h_0}{\phi(i)}\right],$$

which is decreasing in $\phi(i)$. For households with $\phi(i) > \overline{\phi}$, their growth rate $n_0(i)$ would be lower than $n_0(\overline{\phi})$. However, they will have a higher level of human capital in the next period:

$$h_1(i) = \gamma\frac{\phi(i) + (1-\delta)h_0}{1+\eta+\gamma} > \gamma\frac{\overline{\phi} + (1-\delta)h_0}{1+\eta+\gamma}.$$

This higher level of human capital gives rise to a higher growth rate $n_1(i)$ and reduces the difference between $n_1(i)$ and $n_1(\overline{\phi})$. However, as shown in Lemma 1, $n_t(i)$ remains lower than $n_t(\overline{\phi})$ for $\phi(i) > \overline{\phi}$ until $h_t(i)$ converges to its steady-state level in (26) at which point the population growth rate of all households $i \in [0, 1]$ converges to n^* in (28). Therefore, the population growth rates of households with $\phi(i) > \overline{\phi}$ are lower than the population growth rates of households with $\phi(i) < \overline{\phi}$ until $h_t(i)$ converges to its steady-state level in (26). This temporary evolutionary disadvantage of high-ability households will never be compensated despite population trends being equal across households in the long run.

The above analysis implies that there exists a threshold for $\phi(i)$ above (below) which $s^*(i) < 1$ ($s^*(i) > 1$). This in turn implies that[19]

$$\int_0^1 \phi(i)s^*(i)di < \int_0^1 \phi(i)di = \overline{\phi}, \tag{41}$$

because the households with larger $\phi(i)$ end up having a lower steady-state population share $s^*(i)$. Therefore, we also have the following inequality:

$$g^* = \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2}\int_0^1 \phi(i)s^*(i)di - 1 < \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2}\overline{\phi} - 1, \tag{42}$$

where the right-hand side of the inequality is the steady-state innovation-driven growth rate under homogeneous households (i.e., $\phi(i) = \overline{\phi}$ for all $i \in [0, 1]$) in an economy that has experienced the transition to innovation. In other words, the steady-state growth rate g^* becomes lower because the heterogeneity in households and the temporary evolutionary disadvantage of the high-ability households reduce the average level of human capital and consequently the rate of innovation (recall that $g_t = \theta H_{R,t}/L_t$) in the long run. We summarize the above result in the following proposition.

Proposition 1. *The temporary evolutionary disadvantage of the high-ability households causes a lower steady-state equilibrium growth rate g^* than the case of homogeneous households without natural selection.*

Proof. *See Appendix A.* □

4.1 A parametric example

In this section, we provide a simple parametric example to illustrate our results more clearly and prepare for the empirical analysis in Section 5. We consider two types of households. Specifically, $\phi(i) = \overline{\phi} + \varsigma$ for $i \in [0, 0.5]$ and $\phi(j) = \overline{\phi} - \varsigma$ for $j \in [0.5, 1]$. As before, the households own the same initial amount of human capital (i.e., $h_0(i) = h_0$ for $i \in [0, 1]$). Their initial population shares

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are also the same (i.e., $s_0(i) = 1$ for $i \in [0, 1]$); in this case, the mean of $\phi(i)$ is simply $\overline{\phi}$ and the coefficient of variation in $\phi(i)$ is $\varsigma/\overline{\phi}$. Therefore, for a given $\overline{\phi}$, an increase in ς raises the coefficient of variation in $\phi(i)$ and also makes (40) more likely to hold by raising $\int_0^1 [1/\phi(i)]di = 1/(\overline{\phi} - \varsigma^2/\overline{\phi}) > 1/\overline{\phi}$.

From (26), their steady-state levels of human capital are different and given by $h^*(i) = \gamma(\overline{\phi} + \varsigma)/(1 + \eta + \gamma\delta)$ for $i \in [0, 0.5]$ and $h^*(j) = \gamma(\overline{\phi} - \varsigma)/(1 + \eta + \gamma\delta)$ for $j \in [0.5, 1]$. From (42), the steady-state growth rate g^* is given by

$$\begin{aligned} g^* &= \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2}\left[(\overline{\phi}+\varsigma)s_H^* + (\overline{\phi}-\varsigma)s_L^*\right] - 1 \\ &= \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2}\left\{\overline{\phi}+\varsigma\left[s_H^*(\underset{-}{\varsigma}) - s_L^*(\underset{+}{\varsigma})\right]\right\} - 1, \end{aligned} \tag{43}$$

where $s_L^* \equiv \int_{0.5}^1 s^*(j)dj = s^*(j)/2$ is the steady-state population share of household $j \in [0.5, 1]$ with low ability $\phi(j) = \overline{\phi} - \varsigma$ whereas $s_H^* \equiv \int_0^{0.5} s^*(i)di = s^*(i)/2$ is the steady-state population share of household $i \in [0, 0.5]$ with high ability $\phi(i) = \overline{\phi} + \varsigma$. We note that $s_H^* + s_L^* = 1$. Then, from Lemma 1, we have

$$\frac{s_L^*}{s_H^*} = \frac{\prod_{t=0}^{\infty} n_t(j)}{\prod_{t=0}^{\infty} n_t(i)} > 1, \tag{44}$$

where

$$n_t(j) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{\sum_{\tau=0}^{t-1}\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^\tau + \left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^t\left[1+(1-\delta)\frac{h_0}{\overline{\phi}-\varsigma}\right]\right\},$$

$$n_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{\sum_{\tau=0}^{t-1}\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^\tau + \left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^t\left[1+(1-\delta)\frac{h_0}{\overline{\phi}+\varsigma}\right]\right\}.$$

Therefore, s_L^*/s_H^* is increasing in ς, which together with $s_H^* + s_L^* = 1$ implies that s_L^* is increasing in ς and s_H^* is decreasing in ς as stated in (43).

In summary, an increase in ς leads to an immediate increase in the coefficient of variation in $\phi(i)$ given by $\varsigma/\overline{\phi}$ and a subsequent decrease in the steady-state growth rate g^* given by (43) by reducing the average level of human capital and the level of innovation in the long run due to the temporary evolutionary disadvantage of the high-ability households. In the next section, we will test this theoretical prediction using cross-country data.

Corollary 1. *Raising ς causes a larger coefficient of variation in $\phi(i)$ and a lower steady-state growth rate g^*.*

4.1.1 Education policy

We now consider a simple policy experiment. Suppose the government designs a set of policies (which may be public investment in education or an institutional reform in the education system) that improve the schooling system for all households. If these policies are nondiscriminatory, then we can treat them as a proportional shock $\lambda_e > 1$ that scales up the education abilities $\phi(i)$ of all households. High-ability households' ability will become $\lambda_e\left(\overline{\phi} + \varsigma\right)$, whereas low-ability households' ability will become $\lambda_e\left(\overline{\phi} - \varsigma\right)$. Since $\overline{\phi} - \varsigma > 0$, the effects on fertility $n_t(j)$ and $n_t(i)$ are all negative. This result means that education facilities and support will reduce population growth by increasing the family's potential for education. For example, after decades of education policies, China's fertility rate has dropped despite the 2016 abandonment of the single-child policy. Our model allows arguing that China's recent population decline is not easily revertible because the country's fertility transition to quality children is a byproduct of its inclusive and meritocratic education tradition. Will it hamper economic growth? According to our model, it will not. The reader can easily prove that

$$\frac{1+(1-\delta)\frac{h_0}{\lambda_e(\bar{\phi}-\varsigma)}}{1+(1-\delta)\frac{h_0}{\lambda_e(\bar{\phi}+\varsigma)}}$$

decreases in λ_e, which implies - by (44) - that s_L^*/s_H^* decreases as well, thereby leading to an increase in g^* due to the evolutionary process giving rise to a larger population share of high-ability households. Therefore, we can state that:

Corollary 2. *A policy that proportionally raises all education abilities will lead to a decrease in fertility and an increase in long-term economic growth.*

5. EMPIRICAL EVIDENCE

The main theoretical prediction of this study hinges on the quality-quantity tradeoff in fertility, a concept extensively explored by Galor (2005, 2011, 2022) and others in related literature.[20] This tradeoff is rooted in parents' decisions about their children's education: providing education requires time and resources, which limits the number of children they can effectively support. This tradeoff implies that households with higher educational attainment experience an evolutionary disadvantage, leading to a smaller population share over time. This observation is consistent with our model, where households with higher abilities generally possess greater human capital and lower fertility rates, creating a negative relationship between these two endogenous variables.

Corollary 1 of our theoretical model predicts that the negative relationship between fertility and education leads to a detrimental impact of heterogeneity in human capital accumulation on economic growth. To empirically test this prediction, we utilize cross-country data, with global standardized tests of students' academic performance serving as proxies for educational ability. Specifically, we focus on the Trends in International Mathematics and Science Study (TIMSS), a widely recognized assessment of student performance in mathematics and science across the globe. From this data, we calculate the coefficient of variation of scores within each country as a measure of the heterogeneity in educational ability.

TIMSS indicators, being standardized, allow for direct cross-country comparisons.[21] We focus on fourth-grade students, as this group provides a more representative sample of nationwide educational differences compared to ninth-grade students. Prior research by Angrist et al. (2021) has shown that overall student performance varies minimally over time, but significant differences persist across countries. Similarly, the coefficient of variation in TIMSS scores changes little over time but exhibits substantial cross-country variation, making it a robust measure for analyzing the impact of educational ability heterogeneity at the national level.[22]

To address potential omitted variable bias, we include controls for cultural characteristics such as time preference, which could influence both the independent and dependent variables. Specifically, we use the average level of long-term orientation in a country, following Hofstede (1991) and Galor and Özak (2016).[23] We also incorporate a comprehensive set of geographic variables, as suggested by Arbatlı et al. (2020),[24] including factors such as distance from the equator, proximity to waterways, and land suitability for agriculture, all of which have been shown to influence economic development.

Our analysis focuses on 67 countries that participated in the TIMSS tests, examining the impact of educational ability heterogeneity on economic growth from 1951 to 2017. The regression equation is specified as follows:

$$y_{i,t}=\beta_0+\beta_1\text{var}_i+\gamma Z_i+\varphi_t+\varphi_c+\epsilon_{i,t},$$

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Table 1. Heterogeneity of educational ability and economic growth

	Economic Growth			
	(1)	(2)	(3)	(4)
Heterogeneity of educational ability	−8.156***	−7.952***	−13.269***	−9.234***
	(2.484)	(2.279)	(2.787)	(3.366)
Time preference				0.012
				(0.009)
Distance to nearest waterway				0.134
				(0.458)
Absolute latitude				−0.023
				(0.016)
Mean elevation				0.004*
				(0.002)
Mean land suitability				0.782
				(0.561)
Standard deviation of elevation				−0.004**
				(0.002)
Standard deviation of land suitability				0.436
				(1.593)
Island nation dummy				0.114
				(0.573)
Year FE	*N*	*Y*	*Y*	*Y*
Continent FE	*N*	*N*	*Y*	*Y*
R-square	0.009	0.118	0.128	0.136
Observations	3476	3476	3476	3356

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors in parentheses are clustered by country. The dependent variable is the growth rate of GDP per capita.

where (i) $y_{i,t}$ represents economic growth of country i at time t, measured by the annual growth rate of GDP per capita; (ii) the independent variable var_i captures the heterogeneity of educational ability in country i, using the coefficient of variation of TIMSS scores;[25] (iii) Z_i includes controls for time preference and geographical characteristics (e.g., distance to the nearest waterway, absolute latitude, mean elevation, and agricultural land suitability, standard deviation in elevation and agricultural land suitability, and an island dummy);[26] and (iv) φ_t and φ_c are year fixed effects and continent fixed effects, respectively. We provide the summary statistics in Appendix C. In Corollary 1, our theoretical framework predicts that a larger coefficient of variation in ability induces a lower steady-state growth rate, which implies $\beta_1 < 0$.

5.1 Empirical results

Table 1 presents the results from our baseline cross-country analysis. Column 1 starts with a bivariate regression, showing that educational ability heterogeneity is negatively and significantly associated with GDP per capita growth rates. This relationship is visually depicted in Figure 2 of Appendix C, where we plot average annual economic growth rates against educational ability heterogeneity across countries. The scatter plot clearly illustrates a negative relationship, observed across various continents, as indicated by distinct color-coded data points.

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Table 2. Impacts of heterogeneity of educational ability using instrumental variables

	Second stage		First stage	
	Economic Growth		Heterogeneity of educational ability	
	(1)	(2)	(3)	(4)
Heterogeneity of educational ability	−27.314***	−32.498***		
	(9.099)	(10.165)		
Population diversity (ancestry adjusted)			1.432***	
			(0.389)	
Migratory distance from East Africa				−0.011***
				(0.003)
Controls	*Y*	*Y*	*Y*	*Y*
Year FE	*Y*	*Y*	*Y*	*Y*
Continent FE	*Y*	*Y*	*Y*	*Y*
R-square	0.111	0.101	0.738	0.740
Observations	3065	3065	3065	3065
First-stage F statistic			13.064	10.099

Note: $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors in parentheses are clustered by country. The dependent variables in columns 1 and 2 are the growth rate of GDP per capita. Column 1 shows the second-stage results using population diversity (ancestry adjusted) as the instrumental variable, with the corresponding first-stage results presented in Column 3. Column 2 presents the second-stage results using prehistoric migratory distance from East Africa as the instrumental variable, with the corresponding first-stage results shown in Column 4. Controls include time preference, distance to the nearest waterway, absolute latitude, mean elevation, mean agricultural land suitability, standard deviation in elevation, standard deviation in agricultural land suitability, and an island dummy.

To ensure the robustness of our findings, we progressively introduce additional controls. Column 2 adds year fixed effects to account for time-varying factors, while Column 3 incorporates continent fixed effects to control for regional differences. The baseline model presented in Column 4 includes controls for time preferences, geographical characteristics such as proximity to waterways, absolute latitude, mean and standard deviation of elevation, mean and standard deviation of land suitability, and an island dummy.

Despite these controls, concerns about endogeneity remain. The heterogeneity of educational ability and the spatial distribution of economic growth could be jointly influenced by unobserved cultural, institutional, or human factors. For instance, in societies that highly value education, individuals might strive for academic excellence, leading to less variation in test scores and faster economic growth. Ignoring these unobserved factors could bias our results. To address this, we employ two instrumental variables: ancestry adjusted population diversity and migratory distance from East Africa.

Ancestry-adjusted population diversity, as discussed by Ashraf and Galor (2013) and Arbatlı et al. (2020), relates to differences in educational ability and is exogenous to contemporary economic conditions. Migratory distance from East Africa that was used in previous studies, such as Ashraf and Galor (2013), Arbatlı et al. (2020), Ashraf et al. (2021) and Galor et al. (2023), captures the historical migration patterns that shaped genetic diversity, which could influence educational ability heterogeneity.

Table 2 presents the results of the two-stage least squares (2SLS) regressions. In the second stage, Columns 1 and 2 show that the coefficient for educational ability heterogeneity remains significantly negative, confirming its detrimental impact on economic growth. The first-stage results in Columns 3 and 4 validate our instruments, showing a significant positive correlation between educational ability heterogeneity and ancestry adjusted population diversity, and a significant negative correlation with migratory distance from East Africa.

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Table 3. Impacts of heterogeneity of educational ability on education and innovation

	(1) Share of schooling	(2) Average education	(3) R&D	(4) Patent
Heterogeneity of educational ability	−109.662**	−2.080**	−12.572***	−12.686***
	(50.193)	(0.986)	(1.360)	(4.417)
Controls	*Y*	*Y*	*Y*	*Y*
Year FE	*Y*	*Y*	*Y*	*Y*
Continent FE	*Y*	*Y*	*Y*	*Y*
R-square	0.639	0.737	0.761	0.540
Observations	603	603	855	1735

Note: $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors in parentheses are clustered by country. In columns 1–2, the dependent variables are the share of the population with primary schooling and the log of average years of education in the first two columns, respectively. In columns 3–4, the dependent variables are the log of the number of researchers in R&D (per million people) and the log of the number of patent applications, respectively. Controls include time preference, distance to the nearest waterway, absolute latitude, mean elevation, mean agricultural land suitability, standard deviation in elevation and agricultural land suitability, and an island dummy.

Next, we consider education and innovation as alternative proxies for economic growth. In Columns 1 and 2 of Table 3, the dependent variables are the share of the population with at least some primary education and the logarithm of the average years of education, respectively, capturing the average education level. Columns 3 and 4 use the logarithm of the number of researchers in R&D (per million people) and the logarithm of patent applications as dependent variables, representing innovation rates. The negative coefficients for educational ability heterogeneity across these specifications suggest that it adversely affects both education and innovation, further supporting our theoretical predictions.[27] We have also conducted several robustness checks to affirm the reliability of our findings.[28]

6. Conclusion

In this study, we developed an innovation-driven growth model with endogenous takeoff and fertility, offering new insights into the natural selection of heterogeneous households differentiated by their ability to accumulate human capital. Our key findings can be summarized as follows.

In the short run, we demonstrate the emergence of a "survival-of-the-weakest" scenario, where households with higher abilities actually face a temporary evolutionary disadvantage. This disadvantage is eventually mitigated as human capital accumulates. However, in the long run, this temporary setback for high-ability households has enduring negative effects on R&D, technological progress, and long-term economic growth.

Empirically, our cross-country analysis supports the model's predictions, highlighting the detrimental impact of educational heterogeneity on long-term outcomes in education, innovation, and economic growth. While we employ instrumental variables to address potential endogeneity concerns, we acknowledge that these methods may not fully eliminate the issue. Nevertheless, the robustness of our findings across various specifications reinforces the validity of our theoretical predictions.

Theoretically, our work makes a significant contribution by introducing the concept of natural selection among heterogeneous households into the framework of the innovation-driven growth model. This addition enriches the existing literature on economic growth by providing a more nuanced understanding of how demographic dynamics and human capital accumulation interact with innovation processes.

While our model offers valuable insights, it also opens up several avenues for future research. A particularly promising direction would be to explore the policy implications of our findings in

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greater detail. Investigating how government interventions or institutional reforms might influence the interplay between natural selection, human capital accumulation, and economic growth could enhance the practical relevance of our model.

Additionally, integrating other socioeconomic factors—such as cultural attitudes, as explored by Cozzi (1998) and Tabellini (2010), or the dynamics of preference transmission, as discussed by Bisin and Verdier (1998, 2000, 2001, 2017)—could add further depth and realism to the theory. Exploring these extensions could yield valuable insights into how the subtle dynamics of natural selection within heterogeneous households shape macroeconomic outcomes.

In conclusion, our study not only advances the understanding of innovation-driven growth, endogenous takeoff, and natural selection but also raises important questions for future research. By emphasizing the complex interactions between human capital, fertility choices, and economic development, we contribute to a more comprehensive understanding of the multifaceted nature of economic growth.

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Notes

1 https://www.telegraph.co.uk/news/2022/07/06/britons-evolving-poorer-less-well-educated/

2 See also Chu, et al. (2020), Chu, et al. (2020), Iacopetta and Peretto (2021), Chu, et al. (2022), Chu, et al. (2022) and Chu, et al. (2023) for different mechanisms for this endogenous activation of innovation.

3 The formulation is based on Chu, et al. (2016) and Chu, et al. (2022), who however focus on homogeneous households and exogenous fertilty.

4 de laCroix and Doepke (2003) consider a similar utility function by assuming $\eta = \gamma$, such that utility depends on $\gamma \ln [n_t(i)h_{t+1}(i)]$.

5 Our specification differs from de la Croix and Doepke (2003), which in turn is based on Lucas (1988) . In the seminal Lucas model, human capital accumulation alone gives rise to long-run growth, so the addition of technological progress causes exploding growth. In our model, human capital accumulation alone gives rise to a higher level of output in the steady state, whereas long-run growth requires endogenous technological progress driven by innovation.

6 In an OLG setting, Becker et al. (1990) and Blankenau and Simpson (2004) also assume intergenerational transmission of human capital, which is supported by empirical evidence; see Solon (1999) and Black and Devereux (2010) for comprehensive surveys.

7 The quality-quantity tradeoff would still be present if (2) is replaced by $h_{t+1}(i) = \phi(i)e_t(i) + (1-\delta)h_t$, where h_t is the average level of human capital in the society. However, the population would converge to a degenerate distribution, in which households with the lowest $\phi(i)$ would dominate in the long run.

8 Black et al. (2009) provide empirical evidence for a significant intergenerational transmission of IQ scores. See also Jones and Schneider (2006) for data on the variation of average IQ across countries.

9 It is useful to note that $\overline{\phi}$ is the unweighted mean which is exogenous, whereas the weighted mean changes endogenously as the population share of households evolves over time.

10 We follow Yip and Zhang (1997) to specify a linear fertility cost. Palivos (1995) argues that fertility cost may be nonlinear. If we generalize the cost function to $\sigma [n_t(i)]^\chi/\chi$ where $\chi \in (0, \infty)$, our results are robust.

11 In de la Croix and Doepke (2003), childrearing also requires time as an input, but education costs income instead. Our education time cost $e_t(i)$ is equivalent to a reduction in income of $e_t(i)w_t h_t(i)$.

12 Our results are robust to individuals consuming also in the working age.

13 In (5), $e_0(i) = 0$ if $\phi(i) < (1+\eta)(1-\delta)h_0(i)/\gamma$, and $e_t(i) = 0$ until $h_t(i)$ depreciates to a level that reverses this inequality. Then, $e_t(i)$ becomes positive and remains to be so even at the steady state.

14 These differences persist until $h_t(i)$ reaches the steady state.

15 In Appendix B, we discuss the implications of modifying (15) as $\Delta N_t/N_t = \theta H_{R,t}/N_t$.

16 See Laincz and Peretto (2006) for a discussion of the scale effect.

17 See Iacopetta (2010) who considers a model in which innovation occurs before human capital accumulation.

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18 This result is different from Jones (2022) because technological progress depends on population in his model, whereas technological progress depends on human capital per capita in our model.
19 See the proof of Proposition 1 in Appendix A.
20 See for example, Becker et al. (2010) , Fernihough (2017) , and Klemp and Weisdorf (2019) .
21 The program evaluates both fourth and ninth-grade students. Since fourth-grade assessments cover a larger and more representative sample of students, they offer a more accurate depiction of nationwide differences compared to assessments focused solely on ninth-grade students. Therefore, our analysis focuses on fourth-grade students.
22 The coefficient of variation in overall TIMSS scores (math and science combined) exhibited a modest 1.28% change from 2003 to 2019. A similar pattern emerges when analyzing mathematical and scientific ability separately. The coefficient of variation in mathematical ability increased by only 1.6% during this period. For scientific ability, the coefficient of variation increased by a modest 1.91% from 2003 to 2019.
23 We follow Hofstede (1991) and use the average level of long-term orientation among individuals in a country as a proxy for the country's rate of time preference. As highlighted by Galor and Özak (2016) , long-term orientation significantly impacts the formation of human and physical capital, technological advancement, and economic growth.
24 Inspired by Arbatlı et al. (2020), who use cross-sectional data to investigate the impact of population diversity on the annual frequency of new civil conflict onsets, we introduce similar geographic variables due to the following considerations: (i) absolute distance from the equator and proximity to the nearest waterway, which influence economic development through climatological, institutional, and trade-related mechanisms; (ii) geographical isolation, which provides relative immunity from cross-border spillovers; and (iii) variability in land suitability for agriculture and elevation, which has been shown to foster ethnic diversity (Michalopoulos, 2012) and also impacts economic growth through various mechanisms, such as productivity.
25 The TIMSS test has been conducted every four years from 1995 to 2023, with continuous testing of 4th-grade students starting in 2003. Since data for 2023 is not yet available, we utilize test data from 2003, 2007, 2011, 2015, and 2019. To quantify the heterogeneity of educational ability, we compute the coefficient of variation for each country using student scores from each test cycle. Subsequently, we derive the average heterogeneity of educational ability for each country by computing the mean across these test years.
26 The data on time preference is sourced from Galor and Özak (2016) . Data for absolute latitude, mean elevation, standard deviation in elevation, and an island dummy variable are obtained from the Geographically Based Economic Data (G-ECON) project (Nordhaus, 2006). Data for distance to the nearest waterway is from Arbatlı et al. (2020). The land agricultural suitability data is sourced from Michalopoulos (2012) , with the mean or standard deviation at the country level reflecting the average or standard deviation value of the index across the grid cells located within a country's national borders.
27 The share of the population with at least some primary education and the logarithm of the average years of education are calculated from the Barro-Lee educational attainment dataset. The number of researchers in R&D, and the number of patent applications are sourced from the World Bank.
28 First, we use alternative indicators to measure educational ability heterogeneity. TIMSS scores are segmented into Mathematics and Science categories, and we calculate the coefficient of variation for each. We also use years of education as a proxy for educational ability. Second, we introduce additional controls, including institutional factors like legal origins and diversity measures, such as ethnic fractionalization, ethnolinguistic polarization and linguistic fractionalization, using data from Arbatlı et al. (2020). Third, we perform subsample analyses by continent (Africa, Europe, Asia, Oceania, and the Americas) to ensure that our results are not driven by outliers from specific regions. All these results are robust and available upon request.

References

Aghion, P. and P. Howitt. (1992) A model of growth through creative destruction. *Econometrica* 60(2), 323–351.

Alesina, A. F., M. Seror, D. Y. Yang, Y. You and W. Zeng. (2021) Persistence through revolutions. NBER Working Paper 27053, National Bureau of Economic Research, Inc.

Angrist, N., S. Djankov, P. K. Goldberg and H. A. Patrinos. (2021) Measuring human capital using global learning data. *Nature* 592(7854), 403–408.

Arbatlı, C. E., Q. H. Ashraf, O. Galor and M. Klemp. (2020, Diversity and conflict. *Econometrica* 88, 727–797.

Ashraf, Q. and O. Galor. (2011) Dynamics and stagnation in the Malthusian epoch. *American Economic Review* 101(5), 2003–2041.

Ashraf, Q. and O. Galor. (2013) The "Out of Africa" hypothesis, human genetic diversity, and comparative economic development. *American Economic Review* 103(1), 1–46.

Ashraf, Q., O. Galor and M. Klemp. (2021) The ancient origins of the wealth of nations, *Handbook of Historical Economics*, Elsevier, 675–717.

Becker, G., K. Murphy and R. Tamura. (1990) Human capital, fertility, and economic growth. *Journal of Political Economy* 98(5, Part 2), S12–S37.

https://doi.org/10.1017/S1365100525000082 Published online by Cambridge University Press

Becker, S. O., F. Cinnirella and L. Woessmann. (2010) The trade-off between fertility and education: evidence from before the demographic transition. *Journal of Economic Growth* 15(3), 177–204.

Bisin, A. and T. Verdier. (1998) On the cultural transmission of preferences for social status. *Journal of Public Economics* 70(1), 75–97.

Bisin, A. and T. Verdier. (2000) Beyond the melting pot: cultural transmission, marriage, and the evolution of ethnic and religious traits. *Quarterly Journal of Economics* 115(3), 955–988.

Bisin, A. and T. Verdier. (2001) The economics of cultural transmission and the dynamics of preferences. *Journal of Economic Theory* 97(2), 298–319.

Bisin, A. and T. Verdier. (2017) On the joint evolution of culture and institutions. NBER Working Paper 23375

Black, S. E. and P. J. Devereux. (2010) Recent developments in intergenerational mobility. In Black, S. E. and P. J. Devereux. (eds.), *Handbook of Labor Economics*, pp. 1487–1541. Amsterdam: Elsevier, 16

Black, S. E., P. J. Devereux and K. G. Salvanes. (2009) Like father, like son? A note on the intergenerational transmission of IQ scores. *Economics Letters* 105(1), 138–140.

Blankenau, W. and N. Simpson. (2004) Public education expenditures and growth. *Journal of Development Economics* 73(2), 583–605.

Brunnschweiler, C., P. Peretto and S. Valente. (2021) Wealth creation, wealth dilution and demography. *Journal of Monetary Economics* 117, 441–459.

Carillo, M. R., V. Lombardo and A. Zazzaro. (2019) The rise and fall of family firms in the process of development. *Journal of Economic Growth* 24(1), 43–78.

Chu, A., G. Cozzi and C. Liao. (2013) Endogenous fertility and human capital in a Schumpeterian growth model. *Journal of Population Economics* 26(1), 181–202.

Chu, A., H. Fan and X. Wang. (2020) Status-seeking culture and development of capitalism. *Journal of Economic Behavior and Organization* 180, 275–290.

Chu, A., Y. Furukawa and X. Wang. (2022) Rent-seeking government and endogenous takeoff in a Schumpeterian economy. *Journal of Macroeconomics* 72, 103399.

Chu, A., Y. Furukawa and D. Zhu. (2016) Growth and parental preference for education in China. *Journal of Macroeconomics* 49, 192–202.

Chu, A., Z. Kou and X. Wang. (2020) Effects of patents on the transition from stagnation to growth. *Journal of Population Economics* 33(2), 395–411.

Chu, A., Z. Kou and X. Wang. (2022) Culture and stages of economic development. *Economics Letters* 210, 110213.

Chu, A., P. Peretto and X. Wang. (2022) Agricultural revolution and industrialization. *Journal of Development Economics* 158, 102887.

Chu, A., P. Peretto and R. Xu. (2023) Export-led takeoff in a schumpeterian economy. *Journal of International Economics* 145, 103798.

Connolly, M. and P. Peretto. (2003) Industry and the family: two engines of growth. *Journal of Economic Growth* 8(1), 115–148.

Cozzi, G. (1998) Culture as a bubble. *Journal of Political Economy* 106(2), 376–394.

de la Croix, D. and M. Doepke. (2003) Inequality and growth: why differential fertility matters. *American Economic Review* 93(4), 1091–1113.

Fernihough, A. (2017) Human capital and the quantity-quality trade-off during the demographic transition. *Journal of Economic Growth* 22(1), 35–65.

Funke, M. and H. Strulik. (2000) On endogenous growth with physical capital, human capital and product variety. *European Economic Review* 44(3), 491–515.

Galor, O. (2005) From stagnation to growth: unified growth theory. *Handbook of Economic Growth* 1, 171–293.

Galor, O. (2011) *Unified Growth Theory*, Princeton University Press.

Galor, O. (2022) *The Journey of Humanity: The Origins of Wealth and Inequality*, Penguin Random House.

Galor, O., M. Klemp and D. C. Wainstock. (2023) *The Impact of the Prehistoric Out of Africa Migration on Cultural Diversity*, National Bureau of Economic Research.

Galor, O. and S. Michalopoulos. (2012) Evolution and the growth process: natural selection of entrepreneurial traits. *Journal of Economic Theory* 147(2), 759–780.

Galor, O. and O. Moav. (2001) Evolution and growth. *European Economic Review* 45(4-6), 718–729.

Galor, O. and O. Moav. (2002) Natural selection and the origin of economic growth. *Quarterly Journal of Economics* 117(4), 1133–1191.

Galor, O., O. Moav and D. Vollrath. (2009) Inequality in landownership, the emergence of human-capital promoting institutions, and the great divergence. *Review of Economic Studies* 76(1), 143–179.

Galor, O. and A. Mountford. (2008) Trading population for productivity: theory and evidence. *Review of Economic Studies* 75(4), 1143–1179.

Galor, O. and Ö. Özak. (2016) The agricultural origins of time preference. *American Economic Review* 106(10), 3064–3103.

Galor, O. and D. Weil. (2000) Population, technology and growth: from the Malthusian regime to the demographic transition. *American Economic Review* 110(4), 806–828.
Goh, A.-T. and J. Olivier. (2002) Optimal patent protection in a two-sector economy. *International Economic Review* 43(4), 1191–1214.
Grossman, G. and E. Helpman. (1991) Quality ladders in the theory of growth. *Review of Economic Studies* 58(1), 43–61.
Hofstede, G. H. (1991) *Cultures and organizations: software of the mind*. London: McGraw-Hill.
Iacopetta, M. (2010) Phases of economic development and the transitional dynamics of an innovation-education growth model. *European Economic Review* 54(2), 317–330.
Iacopetta, M. and P. Peretto. (2021) Corporate governance and industrialization. *European Economic Review* 135, 103718.
Jones, C. (1995) R&D-based models of economic growth. *Journal of Political Economy* 103(4), 759–784.
Jones, C. (2001) Was an industrial revolution inevitable? Economic growth over the very long run. *The B.E. Journal of Macroeconomics* 1(2), 1–45.
Jones, C. (2022) The end of economic growth? Unintended consequences of a declining population. *American Economic Review* 112(11), 3489–3527.
Jones, G. and W. Schneider. (2006) Intelligence, human capital, and economic growth: a bayesian averaging of classical estimates (BACE) approach. *Journal of Economic Growth* 11(1), 71–93.
Klemp, M. and J. Weisdorf. (2019) Fecundity, fertility and the formation of human capital. *Economic Journal* 129(618), 925–960.
Laincz, C. A. and P. F. Peretto. (2006) Scale effects in endogenous growth theory: An error of aggregation not specification. *Journal of Economic Growth* 11(3), 263–288.
Li, C.-W. (2001) On the policy implications of endogenous technological progress. *Economic Journal* 111(471), C164–C179.
Lucas, R. (1988) On the mechanics of economic development. *Journal of Monetary Economics* 22(1), 3–42.
Michalopoulos, S. (2012) The origins of ethnolinguistic diversity. *American Economic Review* 102(4), 1508–1539.
Nordhaus, W. D. (2006). The 'Stern Review' on the Economics of Climate Change. NBER Working Paper No. 12741.
Palivos, T. (1995) Endogenous fertility, multiple growth paths, and economic convergence. *Journal of Economic Dynamics and Control* 19(8), 1489–1510.
Peretto, P. (2015) From Smith to Schumpeter: a theory of take-off and convergence to sustained growth. *European Economic Review* 78, 1–26.
Peretto, P. and S. Valente. (2015) Growth on a finite planet: resources, technology and population in the long run. *Journal of Economic Growth* 20(3), 305–331.
Romer, P. (1990) Endogenous technological progress. *Journal of Political Economy* 98(5, Part 2), S71–S102.
Segerstrom, P., T. Anant and E. Dinopoulos. (1990) A schumpeterian model of the product life cycle. *American Economic Review* 80, 1077–1091.
Solon, G. (1999) Intergenerational mobility in the labor market. In Solon, G. (eds.), *Handbook of Labor Economics*, pp. 1761–1800, Amsterdam: Elsevier, ch. 29
Tabellini, G. (2010) Culture and institutions: economic development in the regions of Europe. *Journal of the European Economic Association* 8(4), 677–716.
Yip, C. K. and J. Zhang. (1997) A simple endogenous growth model with endogenous fertility: indeterminacy and uniqueness. *Journal of Population Economics* 10(1), 97–110.

Appendix A: Proofs

Proof of Lemma 1. *The labor share of household i is* $s_t(i) \equiv L_t(i)/L_t$, *where*

$$L_t(i) = n_{t-1}(i)L_{t-1}(i) = n_{t-1}(i)n_{t-2}(i)L_{t-2}(i) = \ldots = \prod_{\tau=0}^{t-1} n_\tau(i)L_0(i). \tag{A1}$$

From (4), the fertility choice at time 0 is given by

$$n_0(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left[1 + (1-\delta)\frac{h_0(i)}{\phi(i)}\right]. \tag{A2}$$

From (6), the level of human capital at time 1 is given by

$$h_1(i) = \frac{\gamma\phi(i)}{1+\eta+\gamma}\left[1 + (1-\delta)\frac{h_0(i)}{\phi(i)}\right]. \tag{A3}$$

Substituting (A3) into (4) yields the fertility choice at time 1 as

$$n_1(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{1+\frac{\gamma(1-\delta)}{1+\eta+\gamma}\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\}. \tag{A4}$$

Substituting (A3) into (6) yields the level of human capital at time 2 as

$$h_2(i) = \frac{\gamma\phi(i)}{1+\eta+\gamma}\left\{1+\frac{\gamma(1-\delta)}{1+\eta+\gamma}\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\}. \tag{A5}$$

Substituting (A5) into (4) yields the fertility choice at time 2 as

$$n_2(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{1+\frac{\gamma(1-\delta)}{1+\eta+\gamma}+\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^2\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\}. \tag{A6}$$

Then, we can continue the process to derive the fertility choice at time $t \geq 3$ as

$$n_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{1+\frac{\gamma(1-\delta)}{1+\eta+\gamma}+\ldots+\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^{t-1}+\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^{t}\right.$$
$$\left.\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\}, \tag{A7}$$

which can then be re-expressed using a summation sign as in Lemma 1. □

Proof of Lemma 2. *If (17) holds, then (35) shows that $H_{R,t} > 0$. Now, let's consider the case in which*

$$\int_0^1 [1-e_t(i)-\sigma n_t(i)]\, h_t(i)\frac{L_t(i)}{L_t}di < \frac{1}{\theta}. \tag{A8}$$

Recall that the value of assets at the end of time t must equal the amount of saving at time t given by wage income at time t such that

$$N_{t+1}v_t = w_t\int_0^1 [1-e_t(i)-\sigma n_t(i)]\, h_t(i)L_t(i)di. \tag{A9}$$

Substituting (A9) into (A8) yields

$$w_t > \frac{\theta N_{t+1}v_t}{L_t} \geq \frac{\theta N_t v_t}{L_t}, \tag{A10}$$

where the second inequality uses $N_{t+1} \geq N_t$. Equation (A10) implies that $\Delta N_t v_t = w_t H_{R,t}$ in (16) cannot hold unless $H_{R,t} = 0$. □

Proof of Proposition 1. *From Lemma 1, the steady-state population share of household i is given by*

$$s^*(i) = \frac{\prod_{t=0}^{\infty} n_t(i)}{\int_0^1 \prod_{t=0}^{\infty} n_t(i)di}, \tag{A11}$$

where we have used $L_0(i) = L_0$ for all i. Lemma 1 shows that $n_t(i)$ is monotonically decreasing in $\phi(i)$ before reaching the steady state n^* in (28), which then becomes independent of $\phi(i)$. Therefore, it must be the case that

$$s^*(i) < s^*(j) \Leftrightarrow \phi(i) > \phi(j). \tag{A12}$$

Given that $\int_0^1 s^*(i)di = 1$, there must exist a threshold for $\phi(i)$ above (below) which $s^*(i) < 1$ ($s^*(i) > 1$). Let's define:

$$\Delta \equiv \int_0^1 \phi(i)s^*(i)di - \overline{\phi} = \int_0^1 \phi(i)s^*(i)di - \int_0^1 \phi(i)di = \int_0^1 \phi(i)[s^*(i)-1]di. \tag{A13}$$

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We order the households such that $\phi(i) > \phi(j)$ *for any* $i < j$*. In this case,* $s^*(i) < 1$ *for* $i \in [0, \varepsilon]$ *and* $s^*(i) > 1$ *for* $i \in [\varepsilon, 1]$*. Therefore, we can re-express* Δ *as*

$$\Delta = \underbrace{\int_0^{\varepsilon} \phi(i)[s^*(i) - 1]di}_{<0} + \underbrace{\int_{\varepsilon}^{1} \phi(i)[s^*(i) - 1]di}_{>0}. \tag{A14}$$

If $\phi(i) = \phi(j) = \phi(\varepsilon)$ *for all* $i \in [0, \varepsilon]$ *and* $j \in [\varepsilon, 1]$*, then* $\Delta = 0$ *because*

$$\phi(\varepsilon) \int_0^{\varepsilon} [s^*(i) - 1]di + \phi(\varepsilon) \int_{\varepsilon}^{1} [s^*(i) - 1]di = \phi(\varepsilon) \int_0^{1} [s^*(i) - 1]di = 0. \tag{A15}$$

Otherwise, $\Delta < 0$ *because* $\phi(i) > \phi(\varepsilon) > \phi(j)$ *for any* $i \in [0, \varepsilon)$ *and* $j \in (\varepsilon, 1]$ *such that*

$$\int_0^{\varepsilon} \phi(i)[s^*(i) - 1]di < \phi(\varepsilon) \int_0^{\varepsilon} [s^*(i) - 1]di < 0,$$

$$\phi(\varepsilon) \int_{\varepsilon}^{1} [s^*(i) - 1]di > \int_{\varepsilon}^{1} \phi(i)[s^*(i) - 1]di > 0,$$

implying $\Delta < \phi(\varepsilon) \int_0^1 [s^*(i) - 1]di = 0$*. Therefore, (41) and (42) hold.* □

Appendix B: Alternative innovation specification

In this appendix, we explore the implications of modifying (15) as

$$\frac{\Delta N_t}{N_t} = \frac{\theta H_{R,t}}{N_t}. \tag{B1}$$

In this case, the steady-state growth rate of N_t is determined by the growth rate of $H_{R,t}$ as

$$\frac{\Delta N_t}{N_t} = \frac{\Delta H_{R,t}}{H_{R,t}}. \tag{B2}$$

Recall that the resource constraint on human-capital-embodied labor is given by

$$H_{Y,t} + H_{R,t} = \int_0^1 [1 - e_t(i) - \sigma n_t(i)]\, h_t(i) L_t(i) di, \tag{B3}$$

where $\{e_t(i), n_t(i), h_t(i)\}$ are constant at the steady state and given by (26)-(28). Therefore, the growth rate of $H_{R,t}$ is determined by the growth rate of $L_t(i)$ given by n^*, which is the same across all households. Therefore, the steady-state growth rate of N_t is determined by the population growth rate as in Jones (1995, 2001) and given by

$$g^* = n^* - 1 = \frac{\eta}{\sigma(1 + \eta + \gamma\delta)} - 1, \tag{B4}$$

which is increasing in fertility preference η but decreasing in quality preference γ, human capital depreciation rate δ and fertility cost σ. In this case, household heterogeneity in education abilities $\phi(i)$ does not affect economic growth in the long run. This difference arises because long-run growth is determined by the growth rate of human capital in (B2) in this extended model, rather than the average level of human capital in (15) in the baseline model. Therefore, it becomes an empirical question as to whether the average level of human capital or the growth rate of population is a more relevant determinant of economic growth in the long run.

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Appendix C: Figure and table

Table 4. Summary statistics

Variable	Obs	Mean	S.D.	Min	Max
Growth rate of GDP per capita (%)	3,476	2.481	5.968	−43.743	94.138
Heterogeneity of educational ability (Total)	3,476	0.177	0.070	0.096	0.442
Time preference (%)	3,476	39.676	27.739	13.000	100
Absolute latitude	3,476	36.374	15.163	1.300	64.000
Distance to nearest waterway (km)	3,476	264.034	425.176	14.176	2385.580
Mean elevation (km)	3,476	141.661	226.068	0.003	1096.503
Mean land suitability	3,413	0.436	0.285	0.003	0.954
Standard deviation of elevation	3,476	106.871	171.805	0.000	965.317
Standard deviation of land suitability	3,356	0.185	0.099	0.001	0.387
Island nation dummy	3,476	0.161	0.367	0	1
Population diversity (ancestry adjusted)	3,065	0.723	0.022	0.671	0.746
Migratory distance from East Africa (in 1,000 km)	3,065	8.520	5.915	3.571	25.898
Share of schooling (%)	627	84.422	21.136	5.009	100
Average education	627	1.944	0.550	−0.934	2.586
Number of researchers (logarithm)	893	7.414	1.078	3.108	8.978
Patent application (logarithm)	1,794	6.293	2.532	0	12.859

Notes: The heterogeneity of educational ability is calculated using the coefficient of variation of academic scores from the TIMSS database. The heterogeneity of educational years, share of schooling, and average education data are sourced from the Barro-Lee educational attainment dataset. The data on time preference is sourced from Galor and Ozak (2016) . The land agricultural suitability data is sourced from Michalopoulos (2012) . Data for absolute latitude, mean elevation, standard deviation in elevation, and an island dummy variable are obtained from the Geographically Based Economic Data (G-ECON) project (Nordhaus, 2006). Data for distance to the nearest waterway are from Arbatli et al., (2020) . All other variables are from the Penn World Table.

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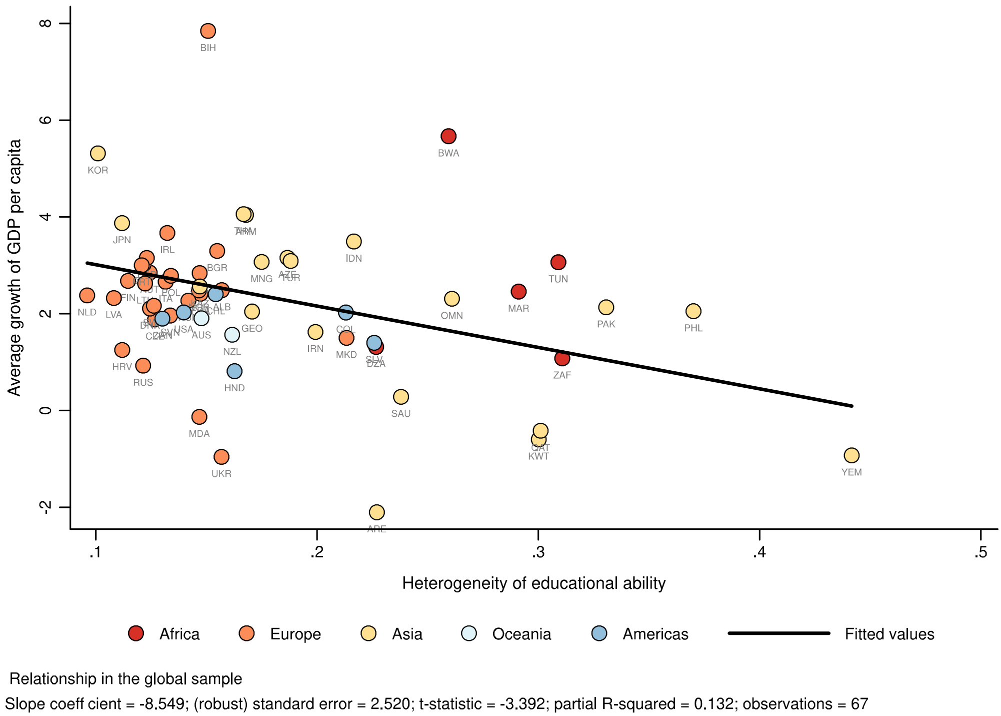

Figure 2. The relationship between heterogeneity in educational ability and economic growth.
Notes: This figure illustrates a negative relationship between heterogeneity in educational ability against the country's average economic growth rate from 1951 to 2017.

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