HELIOSEISMIC INVESTIGATION OF SOLAR INTERNAL STRUCTURE.

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ABSTRACT. The solar oscillation frequencies provide our only means of obtaining detailed information about conditions inside the Sun. Here I give a brief overview of the relevant properties of solar models and solar oscillations, and present examples of the dependence of the oscillation frequencies on the structure of the model. Furthermore I discuss some results obtained so far from analysis of observed frequencies.

1. Introduction.

Observations of solar oscillations have given us a large amount of very precise data on the properties of the solar interior. Recent compilations of observed frequencies (Duvall *et al.* 1988; Libbrecht & Kaufman 1988) list over 2000 frequencies, with estimated errors that are in some cases less than 0.01 per cent. This must be compared with the other observational data that is, or may be, relevant to tests of solar models: the mass, radius and luminosity, all of which are known with comparable precision, and the neutrino flux, which, as is evident from other contributions in this volume, is subject to considerable observational and theoretical uncertainties.

The physical nature and behaviour of the oscillations are in general well understood. The observed modes correspond to standing acoustic waves, or *p modes*. Given a solar model it is relatively straightforward to compute its oscillation frequencies; the details of the behaviour of the oscillations in the uppermost part of the convection zone and the atmosphere are still somewhat uncertain, but the effects of this region can to a large extent be eliminated through suitable analysis of the observations. Apart from this difficulty, the frequencies provide a clean diagnostics of conditions inside the Sun.

What can we hope to learn from these data? An immediate goal is to determine empirically the variation of the relevant properties, in particular the sound speed and perhaps the density, throughout the Sun. Aside from satisfying our curiosity about conditions in the solar interior, this may provide constraints on conditions in the solar core, and hence on the rate of neutrino emission, or lead to determination of the depth of the solar convection zone, which is important to understanding the generation of the solar

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magnetic field (Rädler, these proceedings) and the evolution of the solar surface abundance of, *e.g.* lithium (Baglin & Lebreton, these proceedings).

A more fundamental purpose, however, is to study the basic processes that determine the structure of the solar interior. Computations of stellar evolution are based on assumptions of perhaps questionable validity, and require information, which is often uncertain, about the properties of matter under the conditions in stellar interiors. Analysis of the solar frequencies provides a detailed test of computations of solar models, and may therefore uncover weaknesses in the assumptions that could affect other stellar models. Furthermore, the frequencies are sensitive at a significant level to even quite subtle details of the equation of state or the opacity. Thus it is possible to use the observations to study properties of plasmas under conditions so extreme that they cannot be reproduced in the laboratory.

2. Properties of the solar interior.

It is useful to review very briefly normal calculations of solar models, and their possible shortcomings (see also Bahcall, these proceedings; Turck-Chièze *et al.* 1988; Turck-Chièze 1990). It is assumed that the model is in hydrostatic and thermal equilibrium. Evolution is controlled by the gradual fusion of hydrogen into helium; it is assumed that there is no mixing in the solar interior, so that the composition in any given mass-shell is determined solely by the local nuclear burning. With these assumptions the structure is largely determined by the *microphysics* of the solar interior, *i.e.*

- the equation of state
- the opacity
- the nuclear energy generation rates.

In addition, the computation requires that the solar mass is known, as well as the initial chemical composition, which is assumed to be uniform. The goal is to compute a model at the age of the present Sun, which is also assumed to be known, with the observed radius and surface luminosity.

In practice, the initial helium abundance Y_0 cannot be determined independently and must be regarded as a free parameter of the calculation, as must the "mixing-length" parameter α which measures the efficiency of convective energy transport near the solar surface. Y_0 and α are adjusted until the model of the present Sun has the correct radius and luminosity. In this way one obtains what is sometimes called a "standard solar model". It is evidently dependent on the uncertainties in the assumed microphysics, but is otherwise well-defined.

The equation of state is discussed in these proceedings by Ebeling; furthermore the opacity is discussed in separate papers by Cox and Iglesias. Cox also considers the effects of the opacities on the solar models and their frequencies and predicted neutrino flux. The equation of state must obviously take into account the transition from very little ionization in the solar atmosphere to essentially full ionization in the solar interior. Also thermodynamic consistency must be ensured. However, the implementations used in actual calculations of solar models differ widely in complexity. Among the simplest is the one, in the following referred to as EFF, proposed by

Eggleton, Faulkner & Flannery (1973); this assumes the atoms to be in their ground states and ignores essentially all interactions between the constituents of the gas, but does include a thermodynamically consistent, if largely arbitrary, transition to full ionization in the solar core. At the opposite extreme is the socalled MHD equation of state developed by B. &. D Mihalas, Hummer & Däppen (Hummer & Mihalas 1988; Mihalas, Däppen & Hummer 1988; Däppen *et al.* 1988; see also Däppen 1988). Here the excitation of the atoms is included in considerable detail, and the interactions are described by assigning to each level an occupation probability which depends on the perturbations from other particles. Christensen-Dalsgaard, Däppen & Lebreton (1988) compared models and frequencies computed with these two formulations. I return to this comparison in section 5. Note also that Däppen, Lebreton & Rogers (1990) made a comparison between the MHD equation of state and a conceptually very different formulation (*e.g.* Rogers 1981, 1986).

The opacity is in general obtained from interpolation in tables. Commonly used have been the tables by Cox & Stewart (1970), Cox & Tabor (1976) and, more recently, tables computed with the Los Alamos Opacity Library (Huebner *et al.* 1977). The computation of these tables is complicated by the need to take into account the ionization states and level populations of the atoms responsible for the absorption or scattering of radiation, and to include the effect of large numbers of absorption lines. Differences in the treatment of such effects lead to substantial differences in the computed opacities (Iglesias, Rogers & Wilson 1987; Rozsnyai 1989; Courtaud *et al.* 1990). Thus the opacity is, at least as far as the microphysics is concerned, probably the major source of uncertainty in solar model computations. Particular difficulties may be associated with the opacity in the solar atmosphere. Here recent Los Alamos calculations (*cf.* Cox, Guzik & Kidman 1989) found opacities up to a factor 2 higher than previous values. Below I consider opacities from the Los Alamos Opacity Library supplemented with the new low-temperature opacities (in the following LAOL) and compare with results obtained with the Cox & Tabor tables (CT).

The computation of "standard" solar models ignores, or grossly simplifies, a number of processes that might be labelled the *macrophysics* of the Sun. These include

- energy transport
- dynamics of convection
- convective overshoot
- molecular diffusion
- core mixing
- magnetic fields

Energy transport by radiation is treated adequately in the solar interior in the diffusion approximation; on the other hand energy transport by convection is treated in a rather crude way, with furthermore depends on the *a priori* unknown parameter α. Near the surface convection is probably sufficiently vigorous to have dynamic effects on the average hydrostatic equilibrium, yet such effects are often ignored. At the lower boundary of the convection zone motion is normally supposed to stop at the point where convective instability ceases; there is no doubt, however, that motion extends into the convectively stable region, through convective overshoot, although the extent of the overshoot is uncertain. Molecular diffusion is likely to have some effect on the composition profile in the convectively stable region, yet with a few exceptions has been ignored. Instabilities in the deep interior could lead to material mixing, affecting the composition profile and hence solar evolution. (Note that mixing in the solar interior is reviewed in the papers by Spruit and by Zahn, these proceedings). Finally, magnetic fields dominate the structure of the upper solar atmosphere and may have some effect at the photospheric level. The nature or strength of the subphotospheric field is unknown, but one probably cannot totally exclude a field of sufficient magnitude to have an effect on the overall structure of the Sun.

Despite the complications it introduces, convection in a certain sense simplifies the structure of the outer parts of the Sun. Regardless of the uncertain details of convective energy transport, there is no doubt that except in a thin boundary layer near its top the convection zone is very nearly adiabatically stratified, so that gradient of density ρ is given by

$$\frac{d\ln\rho}{dr} \approx \frac{1}{\Gamma_1}\frac{d\ln p}{dr}, \tag{2.1}$$

where r is distance from the centre, p is pressure, and $\Gamma_1 = (\partial\ln p/\partial\ln\rho)_s$, the derivative being at constant specific entropy s. The structure of the adiabatic part of the convection zone is determined by this relation, together with the equation of hydrostatic support. Hence it only depends on the equation of state, the composition and the constant value of the specific entropy, which in turn is essentially fixed by the value of α; in particular, the convection zone structure is insensitive to the opacity.

It should also be noted that much of the uncertain macrophysics is concentrated very near the surface. This is true of the dynamical effects of convection, since convective velocities are likely to be very small elsewhere, of the details of convective energy transport, and of the effects of the visible magnetic field. Apart from convective overshoot and a hypothetical strong internal magnetic field, the remaining difficulties listed are concerned with the composition profile in the radiative interior of the model. Although the list of problems is not exhaustive, this argument gives some support to the simplified view of solar structure shown in Figure 1.

Quite apart from the uncertainties in the physics, it is important to consider with sufficient care the *numerical accuracy* in the computation of solar models. To utilize fully the precision of the observed frequencies to study the properties of the solar interior, we must require that the error in the calculation, given the physics, is no greater than the observational error. This is a far more stringent requirement than is normally imposed on stellar evolution calculations. In an attempt to meet it, a collaboration has been set up under the GONG project (Hill, these proceedings) to compare independently computed solar models with precisely defined physics. Some initial results were reported by Morel, Provost & Berthomieu (1990). In one case, the differences between pressure, density and sound speed in two independent models of the present Sun have been reduced to below 0.06 per cent. This is encouraging, although it still does not quite meet the precision of the most accurately determined observed frequencies.

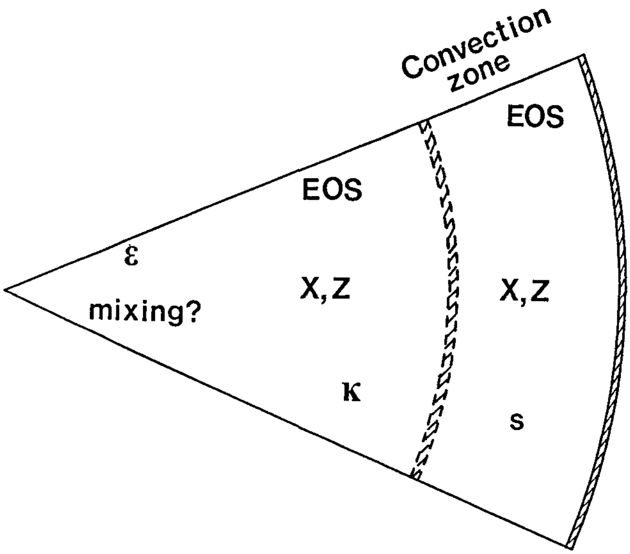

Figure 1. Schematic representation of solar structure. The thin hashed area near the surface indicates the region where the physics is uncertain, because of effects of convection, nonadiabaticity, *etc.* At the base of the convection zone, convective overshoot introduces additional uncertainty. The structure of the adiabatic part of the convection zone is determined by the equation of state (EOS), and the constant values of specific entropy s, and composition (given by the abundances X and Z of hydrogen and heavy elements). Beneath the convection zone the structure also depends on opacity κ and the energy generation rate ε.

3. Properties of solar oscillations.

A mode of oscillation of the Sun is characterized by three wave numbers: the *radial order* n which, roughly, gives the number of zeros in the eigenfunction in the radial direction; the *degree* ℓ; and the *azimuthal order* m, ranging between $-\ell$ and ℓ, which measures the number of zeros in longitude. The degree is related to the horizontal wavenumber k_h and wavelength λ of the mode at radius r by

$$k_h = \frac{2\pi}{\lambda} = \frac{L}{r}, \tag{3.1}$$

where $L = \sqrt{\ell(\ell+1)}$.

Apart from damping or excitation, the time dependence of a single mode is harmonic, as $\cos(\omega t)$. In general the *angular frequency* $\omega = \omega_{n\ell m}$ depends on all three wave numbers. However, if rotation or other departures from spherical symmetry are ignored, $\omega_{n\ell m}$ does not depend on m. This follows from the fact that in this case there is no preferred axis in the star; since m depends on the choice of coordinate axis, the physics of the oscillations, and hence their frequencies, must be independent of m. I shall adopt this approximation here; it should be noted in passing, however, that the m-

dependence of the frequencies permit studies of solar internal rotation (*e.g.* Duvall *et al.* 1984; Brown *et al.* 1989). - In addition to ω, the *cyclic frequency* $\nu = \omega/(2\pi) = 1/P$, is commonly used, particularly in discussions of observed frequencies; here P is the oscillation period.

In calculations of solar oscillation frequencies it is common to ignore a number of complicating features that are so far badly understood, such as

- nonadiabaticity
- excitation, more generally
- dynamical effects of convection
- detailed atmospheric behaviour
- magnetic fields.

These approximations are in some sense similar to those underlying the computation of standard solar models. Calculations that do take into account some of the features (*e.g.* Christensen-Dalsgaard & Frandsen 1983; Kidman & Cox 1984; Balmforth & Gough 1988, 1990) show that they may change the frequencies by several μHz. Thus they have a substantial effect on comparisons between observed and computed frequencies. On the other hand, the complications are all (again with the possible exception of a very strong deep-seated magnetic field) located near the solar surface. Thus they add to the uncertainty of the surface region indicated in Figure 1 but do not directly affect the properties of the oscillations in the deeper solar interior.

With this simplification the computation of the oscillation frequencies is a straightforward numerical problem. Nevertheless, some care is evidently needed to obtain sufficient precision, particularly in view of the fact that the radial order of some of the observed modes is high.

As an aid to understanding the results of the numerical calculations, and to interpret the observations, asymptotic theory has been very useful. The p modes can be approximated locally by plane sound waves, with the dispersion relation $k^2 \equiv k_r^2 + k_h^2 = \omega^2/c^2$. Here k_r and k_h are the radial and horizontal components of the wave vector, and c is the adiabatic sound speed. For a mode of oscillation, k_h is given by equation (3.1), to that

$$k_r^2 = \frac{\omega^2}{c^2} - \frac{L^2}{r^2}. \tag{3.2}$$

Close to the surface, c is small and hence k_r is large. Here the modes propagate almost vertically. With increasing depth, c increases and k_r decreases, until the point is reached where $k_r = 0$ and the wave propagates horizontally. The location $r = r_t$ of this *turning point* is determined by

$$\frac{c(r_t)}{r_t} = \frac{\omega}{L}. \tag{3.3}$$

It corresponds to a point of total internal reflection; for $r < r_t$, $k_r^2 < 0$, and the mode decays exponentially. The behaviour at the surface requires a more careful analysis, which shows that below a critical cut-off frequency (which in the solar atmosphere corresponds to a cyclic frequency of about 5200 μHz) the wave is reflected by the steep density gradient. Thus the wave propagates in a series of "bounces" between the surface and the turning point. A mode of oscillation is a standing wave, formed as an interference pattern between such bouncing waves. It is trapped between the surface and r_t, and hence its frequency depends largely on conditions in this region.

Due to the rapid decrease of the sound speed with increasing radius, the first term on the right hand side of equation (3.2) is substantially larger than the second except near or below the turning point. Thus except near their turning points modes of the same frequency but different degree have essentially the same k_r; thus the properties of the modes, and their response to solar structure, are similar.

Figure 2 illustrates r_t as calculated from equation (3.3). Modes at highest observed values of ℓ are confined to the outermost fraction of a percent of the solar radius, whereas the lowest-degree modes penetrate essentially to the centre.

This simple description of the p modes may be extended to give an asymptotic relation for their frequencies (Gough 1984; Christensen-Dalsgaard *et al.* 1985): The condition for a standing wave is that the change in phase in the radial direction is an integral multiple of π, apart from a contribution which takes into account the phase change at the inner turning point and at the surface. This condition may be expressed as

$$\int_{r_t}^{R} k_r dr \approx (n + \alpha)\pi, \quad \text{or} \quad \int_{r_t}^{R} \left[1 - \left[\frac{Lc}{\omega r}\right]^2\right]^{1/2} \frac{dr}{c} \approx \frac{\pi(n + \alpha)}{\omega}, \qquad (3.4)$$

where we used equation (3.2); here α is the quantity which takes into

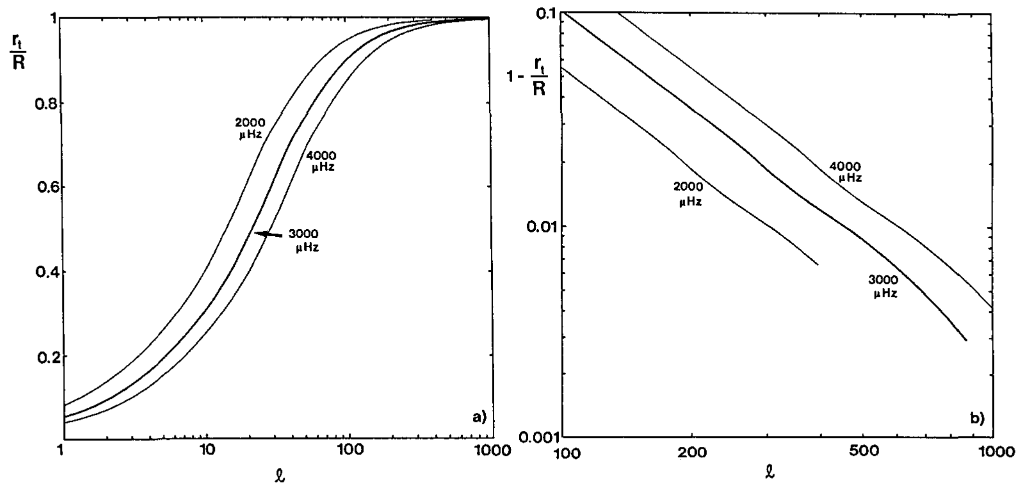

Figure 2. The turning point radius r_t (a) and the penetration depth $R - r_t$ (b), in units of the solar radius R, as a function of degree ℓ for three values of the frequency ν. This has been calculated from equation (3.3) for a model of the present Sun.

account the phase change at the reflection points and hence in particular depends on conditions near the surface. From equation (3.3) follows that the left hand side of equation (3.4) is a function $F(\omega/L)$ of ω/L. Thus this equation establishes a very particular relation among the p-mode frequencies. That solar oscillations satisfy such a relation was first found by Duvall (1982) from observed frequencies.

For small ℓ, equation (3.4), with L replaced by $\ell + \frac{1}{2}$, reduces to

$$\nu \sim (n + \frac{\ell}{2} + \frac{1}{4} + \alpha)\Delta\nu \tag{3.5}$$

where

$$\Delta\nu = \left[2\int_0^R \frac{dr}{c}\right]^{-1} \tag{3.6}$$

is the inverse of twice the sound travel time between the centre and the surface (*e.g.* Tassoul 1980). Thus there is approximately a uniform spacing $\Delta\nu$ between modes of same degree, but different order. Equation (3.5) also predicts the approximate equality $\nu_{n\ell} \approx \nu_{n-1,\ell+2}$. This frequency pattern has been observed for the solar 5 min modes of low degree and may be used in the search for stellar oscillations of solar type.

It is of great interest to consider the *deviations* from this simple relation. The separation $\delta\nu_{n\ell} = \nu_{n\ell} - \nu_{n-1,\ell+2}$ is predominantly determined by conditions in the solar core (*e.g.* Provost 1984; Gough 1986a), since, as argued above, only here does the behaviour of the modes depend substantially on ℓ. A more careful analysis shows that the average separation satisfies

$$\langle\delta\nu_{n\ell}\rangle \approx (4\ell + 6)D_0. \tag{3.7}$$

In section 5 I consider examples of the dependence of the constant D_0 on the structure of the solar core.

By linearizing equation (3.4) one may obtain an expression for the frequency change $\delta\omega$ caused by a change in the solar model, with resulting changes δc and $\delta\alpha$ in c and α (Christensen-Dalsgaard, Gough & Pêrez Hernández 1988). Since c/r decreases quite rapidly with increasing r, $L^2c^2/r^2\omega^2 \ll 1$ except near the turning point r_t, and as a first approximation may be neglected in the resulting expression. If furthermore the term in $\delta\alpha$ can be neglected, the result is the very simple relation between the changes in sound speed and frequency:

$$\frac{\delta\omega}{\omega} \approx \frac{\int_{r_t}^R \frac{\delta c}{c}\frac{dr}{c}}{\int_{r_t}^R \frac{dr}{c}} \tag{3.8}$$

This shows that the change in sound speed in a region of the Sun affects the frequency with a weight determined by the time spent by the mode, regarded as a superposition of traveling waves, in that region. Thus changes near the surface, where the sound speed is low, have relatively large effects on the frequencies. Although this expression is only a rough approximation, it is a useful guide in attempts to interpret frequency differences between models, or between observed and computed frequencies. Note that according to equation (3.8) $\delta\omega/\omega$ depends on the properties of the mode only through

r_t, which in turn is determined by ω/L (*cf.* equation (3.3)).

From a physical point of view, the denominator in equation (3.8) corrects for the fact that with increasing r_t the modes extend over a smaller fraction of the solar mass, and hence their frequencies are easier to perturb. A similar result is obtained from a perturbation analysis, based on the exact oscillation equations, of the effects of modifications to the model or the physics of the oscillations (*e.g.* Christensen-Dalsgaard 1988a; Christensen-Dalsgaard & Berthomieu 1990). In analyses of frequency differences between models, or between observations and theory, this effect may be eliminated by considering scaled frequency differences $Q_{n\ell}\delta\omega_{n\ell}$. Here

$$Q_{n\ell} = \frac{E_{n\ell}}{\bar{E}_0(\omega_{n\ell})}, \qquad (3.9)$$

where $E_{n\ell}$ is a measure of the inertia in the mode, integrated over the volume of the Sun, and $\bar{E}_0(\omega)$ is the value of $E_{n\ell}$ for $\ell = 0$, interpolated to the frequency ω. Roughly speaking, $Q_{n\ell}\delta\omega_{n\ell}$ measures the effect of the part of the modification which is confined to the region where the actual mode is trapped, on a radial mode of the same frequency.

As argued above, near the surface the behaviour of the oscillations depends on frequency but not on ℓ. Thus, if the modification is confined close to the surface its effect on the frequency, when corrected for the ℓ-dependence of the mode inertia, is a function of frequency alone; so therefore is $Q_{n\ell}\delta\omega_{n\ell}$. The condition for this to be true is that the extent of the region over which the modification is significant is much smaller than the depth of penetration of the modes considered. It follows that if $Q_{n\ell}\delta\omega_{n\ell}$ *does* depend on ℓ for a set of modes, the change in the model extends at least to the lower turning point of those modes.

The upper reflection of the modes occurs at increasing depth with decreasing frequency; thus the mode amplitude very near the surface, relative to the amplitude in the interior, decreases (Libbrecht 1988; Christensen-Dalsgaard 1988b). It follows that low-frequency modes are insensitive to modifications that are confined to the superficial layers of the model.

These properties of the oscillations are particularly important in the light of the currently unavoidable errors near the surface of the model. These errors may be expected to lead to scaled frequency errors that are essentially independent of ℓ and small at low frequency. Frequency errors that do not have these properties therefore indicate errors in the bulk of the model.

4. Sensitivity of the frequencies to changes in solar structure.

To provide an illustration of the principles discussed in the previous section, it is interesting to consider specific examples of how a solar model and its frequencies respond to changes in the physics. For p modes, which are the only modes considered here, it follows from section 3 that the change in the sound speed is particularly relevant. A detailed discussion of the effects of various modifications was presented by Christensen-Dalsgaard (1988a). Cox *et al.* (1989) studied the effects of molecular diffusion, Christensen-

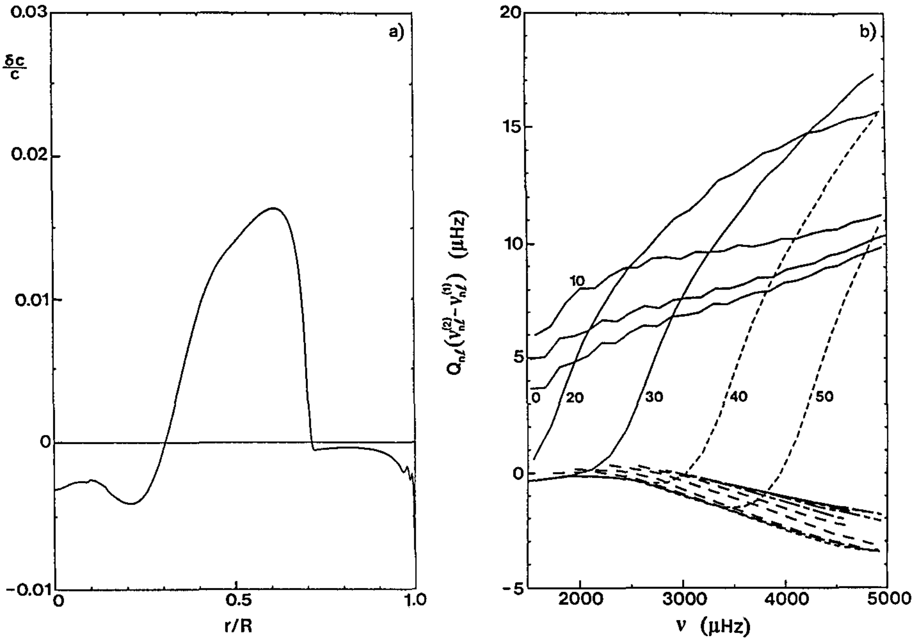

Figure 3. (a) Sound-speed difference, at fixed fractional radius r/R, between the model with modified opacity in the interior and the reference model, in the sense (modified model) - (reference model). (b) Frequency differences between the same two models. The differences have been scaled by the inertia ratio $Q_{n\ell}$ (*cf.* equation (3.9)). Points corresponding to a given value of ℓ have been connected, according to the following convention: ℓ = 0, 5, 10, 20, 30 (————); ℓ = 40, 50, 70, 100 (- - - - - - - - - -); ℓ = 150, 200, 300, 400 (— — — — — —); and ℓ = 500, 600, 700, 800, 900, 1000 (— - — - — - -). In addition a few values of ℓ have been indicated in the figure.

Dalsgaard, Däppen & Lebreton (1988) compared frequencies computed with different assumptions about the equation of state, and Gough & Novotny (1990) studied the effect of the assumed solar age. An interesting analysis of the effects of the opacity was presented by Korzennik & Ulrich (1989).

Here I concentrate on effects of modifications to the opacity. Indeed, it was argued in section 2 that apart from the very uncertain region near the solar surface, the opacity is probably the least well-determined of the physical properties required to compute a model of the Sun. The results presented here were discussed in considerably more detail by Christensen-Dalsgaard & Berthomieu (1990).

All models were calibrated to have the solar radius and luminosity. The reference model was similar to model 1 of Christensen-Dalsgaard (1982), although it was computed with considerably better numerical precision. To obtain the modified models, the opacity κ was changed by adding to $\log\kappa$ a function of temperature which was only different from zero in a restricted temperature interval. *I first consider the effect of a modification in the* opacity near the base of the convection zone, the maximum change in opacity at fixed T and ρ being about 26 per cent. In Figure 3a is shown the resulting change in the sound speed. A striking feature is the comparatively small change in much of the convection zone. In fact it is easy to show that except in the outer part of the convection zone the sound speed is approximately determined by the total mass and surface radius of the model, which are the same in the two cases (*e.g.* Christensen-Dalsgaard 1986). The opacity increase caused an increase in the depth of the convection zone by about 0.02 R. As a result, the temperature gradient is higher (being adiabatic) in the modified model than in the reference model just beneath the bottom of the convection zone of the latter. This is the reason for the increase in $\delta c/c$ with increasing depth just beneath the convection zone.

Frequency differences between the modified and the reference model are shown in Figure 3b. In accordance with the discussion in section 3, the differences have been scaled by the ratio $Q_{n\ell}$ of mode inertias (*cf.* equation (3.9)); for the highest-degree modes the raw differences are larger by a factor of about 5 than those shown. These differences can be understood relatively simply in terms of the differences in sound speed shown in Figure 3a and the behaviour of the turning point illustrated in Figure 2. At very low degree the modes penetrate essentially to the centre, and the frequency change is given by the weighted average in equation (3.8), which is dominated by the region of positive δc beneath the convection zone. As ℓ increases to 10 the turning point moves out through the region of slightly negative δc near the core, and $Q_{n\ell}\delta\nu_{n\ell}$ increases. At higher degrees,

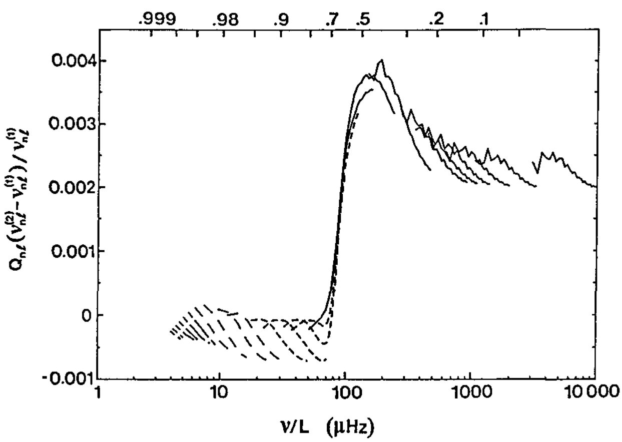

Figure 4. Scaled relative frequency differences corresponding to the differences in Figure 3b, but plotted against $\nu/(\ell + \frac{1}{2})$. The upper axis is labelled in terms of the corresponding turning point position r_t/R. The tick marks are at r_t/R = 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.98, 0.99, 0.995 and 0.999. The same coding of the lines is used as in Figure 3b.

beginning at low frequency for $\ell = 20$, and at increasing ν when ℓ increases to 50, the modes become largely confined within the convection zone, where δc is negative; thus the frequency differences are negative. It should be noticed that the negative δc near the surface has a substantial effect on the frequencies, despite its insignificant appearance in Figure 3a. The reason is the weighting with c^{-1} (*cf.* equation (3.8)) which makes the frequencies very sensitive to changes in the model near the surface.

It follows from this description that the behaviour of the frequency differences can to a large extent be described in terms of the turning point position r_t. This is seen more clearly in Figure 4, where $Q_{n\ell}\delta\nu_{n\ell}/\nu_{n\ell}$ is plotted against $\nu/(\ell + \frac{1}{2})$ which according the equation (3.3) determines r_t (it follows from a more careful asymptotic analysis that L in equation (3.2) should be replaced by $\ell + \frac{1}{2}$). Particularly striking is the transition near $\nu/(\ell + \frac{1}{2}) = 100$, where the turning point moves from beneath to above the base of the convection zone, and the modes therefore no longer penetrate into the region of positive $\delta c/c$.

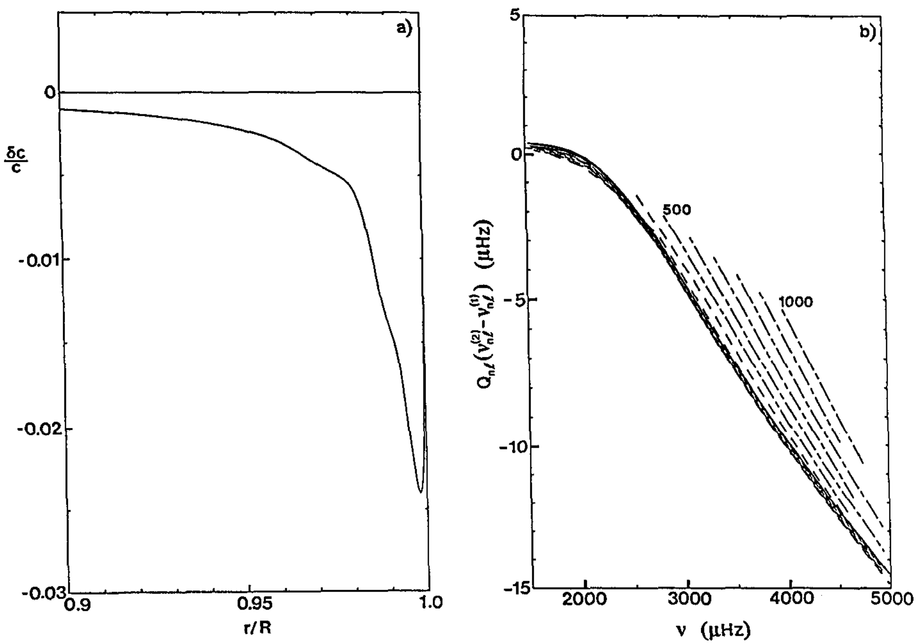

Figure 5. (a) Sound-speed difference, at fixed fractional radius r/R, between a model of the present Sun where the opacity has been artificially increased by up to about a factor 2 near the surface and a normal model, in the sense (modified model) - (reference model). (b) Scaled frequency differences between the same two models. Points corresponding to a given value of ℓ have been connected, according to the same convention as in Figure 3b.

To investigate the effects of the recently suggested opacity increase in the solar photosphere (*cf.* section 2), I computed a model where $\log_{10}\kappa$ was increased by 0.3 in the atmosphere and the upper part of the convection zone. Figure 5a shows the resulting sound-speed difference between the models, in the outer part of the convection zone. The change in the deeper parts of the model is essentially negligible. Corresponding scaled frequency differences are illustrated in Figure 5b. Modes of degree less than about 500 all penetrate well beyond the region where the sound speed is affected (*cf.* Figure 2), and for these the scaled frequency change is mainly a function of frequency, but depends little on the depth of penetration of the mode, and hence on ℓ. At higher degree the modes sample only part of the negative sound speed difference, and the frequency change is smaller.

Quite apart from the specific modification considered, this example illustrates the important point, discussed in section 3, that changes near the surface cause scaled frequency changes that depend mainly on frequency and are small at low frequency. Qualitatively similar changes result from modifications to the treatment of the upper, significantly superadiabatic part of the convection zone (Christensen-Dalsgaard 1986). More generally, it seems likely that the uncertainties in the treatment of the surface layers (nonadiabaticity of the oscillations; the treatment of convection; possible effects of magnetic fields; *etc.*) would have a similar effect on the frequencies. Thus in analyzing observed frequencies it is possible to absorb these uncertainties by allowing an undetermined frequency-dependent part of the scaled differences between observed and computed frequencies.

5. Comparison with observed frequencies.

It is of obvious interest to compare observed frequencies of solar oscillations with frequencies of representative models. Here I consider 4 such models, differing in the treatment of the equation of state or the opacity, as discussed in section 2. The equation of state was obtained either from the EFF or the MHD formulation. The opacity was obtained from interpolation in either the CT or the LAOL tables. Finally the nuclear reaction parameters were essentially as in Bahcall & Ulrich (1988). In the following the models will be labelled as, for example, (EFF, CT) for the model with the Eggleton *et al.* equation of state and the Cox & Tabor opacity table.

Figure 6 shows differences between selected observed frequencies from Duvall *et al.* (1988) and Libbrecht & Kaufman (1988), and computed frequencies for the (EFF, CT) model. The dominant feature in this plot is the increase in the magnitude of the differences with increasing degree. However, this is largely caused by the variation in the mode inertia due to the decrease of the extent of the region where the modes are trapped. If instead one considers the scaled frequency differences $Q_{n\ell}\delta\nu_{n\ell}$, shown in Figure 7a, most of the ℓ-dependence is eliminated. As argued in section 3, this suggests that the dominant errors in the model or the frequency calculation is located close to the solar surface.

It is interesting to analyze in more detail these scaled differences, as well as those for the other three combinations of equation of state and opacity which are also shown in Figure 7. A closer look at Figure 7a shows

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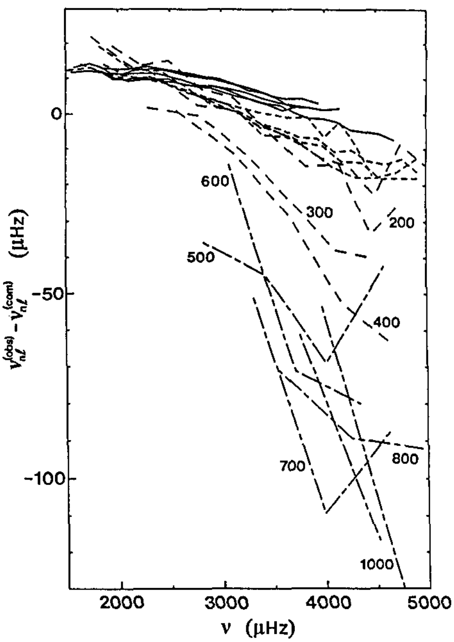
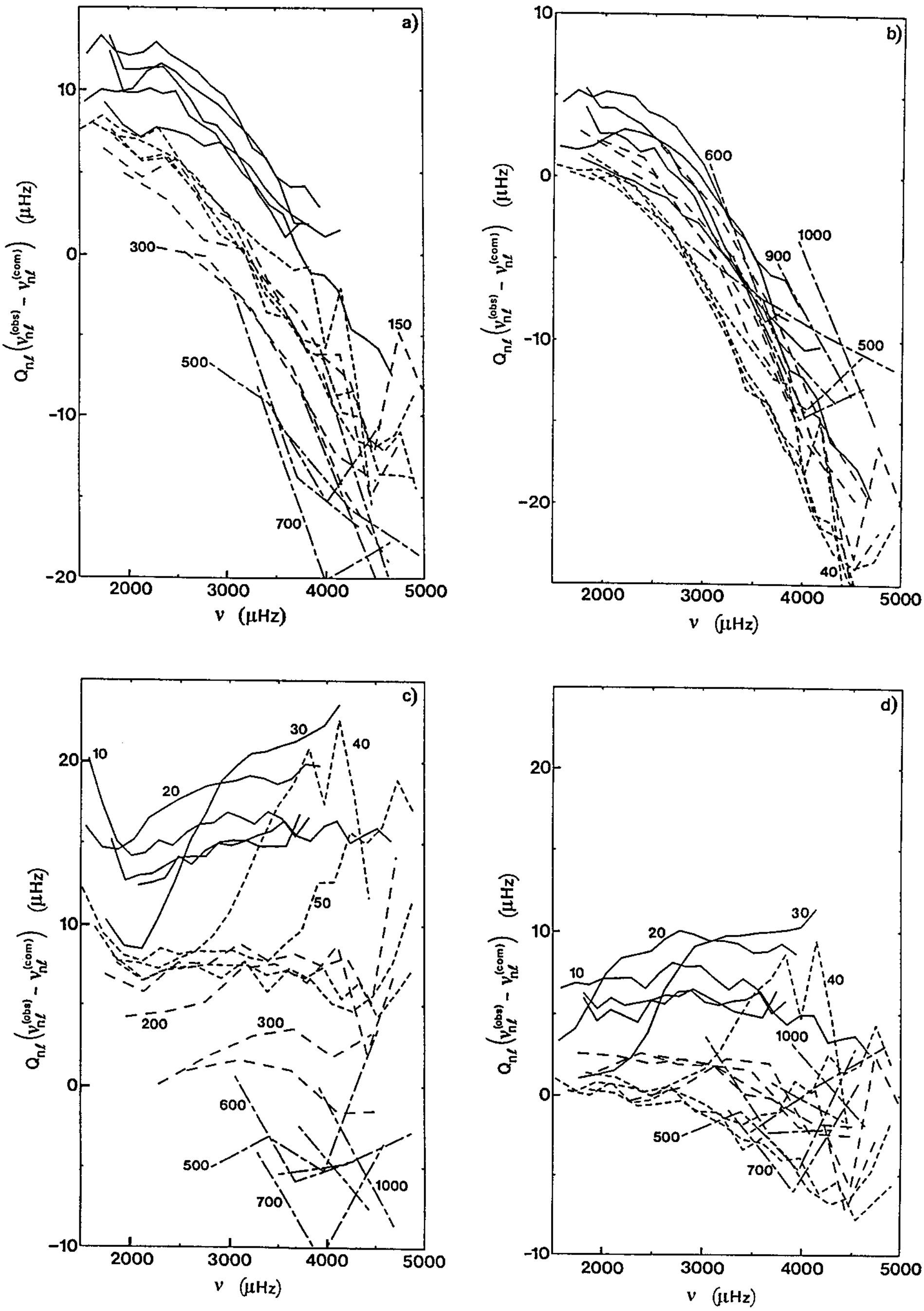


Figure 6. Differences between observed frequencies and frequencies for a model computed with the EFF equation of state and the CT opacity tables. Points corresponding to a given value of ℓ have been connected, according to the same convention as in Figure 3b.

Figure 7 (following page). Scaled differences between observed frequencies and frequencies computed for models of the present Sun. The observed data are from Duvall *et al.* (1988) and Libbrecht and Kaufman (1988). As described in the text, the models differ in the choice of equation of state and opacity tables as follows: a) EFF equation of state, CT opacities; b) MHD equation of state, CT opacities; c) EFF equation of state, LAOL opacities; d) MHD equation of state, LAOL opacities. Points corresponding to a given value of ℓ have been connected, according to the same convention as in Figure 3b.


a)
10
0
-10
-20
2000
3000
4000
5000
ν (μHz)
300
150
500
700
b)
10
0
-10
-20
600
1000
900
500
40
2000
3000
4000
5000
ν (μHz)
c)
20
10
0
-10
10
20
30
40
50
200
300
600
500
1000
700
2000
3000
4000
5000
ν (μHz)
d)
20
10
0
-10
30
20
10
40
1000
500
700
2000
3000
4000
5000
ν (μHz)

that there is still a systematic variation with ℓ in $Q_{n\ell}\delta\nu_{n\ell}$. This is visible as a shift between ℓ = 20 and 40, and a gradual increase in the magnitude of the differences at higher ℓ. Also, there are substantial differences at low frequency. Both these features indicate that there are significant errors in the interior of the model. If the EFF equation of state is replaced by the MHD formulation (*cf.* Figure 7b), the differences at low frequency are reduced substantially, as is the ℓ-dependence. This strongly indicates that the error in the interior of the model, particularly in the convection zone where modes of degree higher than about 50 are trapped, has been reduced by the introduction of the MHD equation of state (Christensen-Dalsgaard, Däppen & Lebreton 1988). The dominant remaining trend is the strong frequency dependence of the differences, as well as a barely visible shift around ℓ = 40. The latter feature must be associated with errors in the model around the turning point of modes of degree 40, *i.e.* at or below the base of the convection zone (*e.g.* Christensen-Dalsgaard & Gough 1984). The presence of a frequency-dependent part of the differences could have been expected, given the approximations in the calculation. The neglect of nonadiabaticity and the effects of convection could well introduce scaled frequency errors of this magnitude, and, as argued in section 3, they would be expected to be largely independent of ℓ.

In the (EFF, LAOL) model (Figure 7c) the variation in $Q_{n\ell}\delta\nu_{n\ell}$ with ℓ is increased substantially over the (EFF, CT) model. Again the use of the MHD equation of state (Figure 7d) causes a marked decrease in the variation for modes trapped in the convection zone, confirming the earlier conclusion that MHD gives the better representation of the equation of state. For modes penetrating beneath the convection zone, however, there remains a substantial ℓ-dependence, indicating that the use of the LAOL opacities increases the errors in the radiative interior of the model. Indeed, although the LAOL opacities are generally somewhat larger than the CT values, near the base of the convection zone they are about 10 per cent smaller, causing the depth of the convection zone to be smaller in the LAOL models. Thus, for example, the depth is 0.283 R in the (MHD, CT) model and 0.267 R in the (MHD, LAOL) model. A more careful analysis of the oscillation frequencies shows that in the Sun the depth of the convection zone is close to the former value (Christensen-Dalsgaard, Gough & Thompson 1990) The difference in convection zone depth causes a pattern of frequency differences, when going from modes penetrating beneath to modes trapped within the convection zone that is qualitatively similar to what was observed in Figure 3b. That the LAOL opacities may be too low in this region was also noticed by Cox *et al.* (1989) and Korzennik & Ulrich (1989).

The most striking effect of using the LAOL opacities, however, is that the frequency-dependent part of the differences has been reduced substantially. This is most evident in Figure 7d, for the (MHD, LAOL) model. For this model the errors, for modes of degree exceeding 100, are comparable with the estimated observational errors. The effect of using the LAOL opacity on this part of the differences is caused by the opacity increase at low temperature. The effect of such an increase was presented in Figure 5b and is comparable with the differences in the (MHD, CT) case shown in Figure 7b. Some care is required, however, when interpreting this result. It might be tempting to take the agreement between observation and theory at face value, as an indication that the LAOL opacities are to be preferred at low temperature, and that there is then little remaining error in the description of the model and the oscillations near the surface. Such a conclusion would be premature, given the known inadequacies in the frequency computation. It is likely that the agreement in Figure 7d is fortuitous, resulting from a partial cancellation of several sources of error.

It was argued in section 3 that for low-degree modes the quantity D_0, which is related to the average frequency difference $\langle \nu_{n\ell} - \nu_{n-1\,\ell+2} \rangle$ by equation (3.7), is a measure of conditions in the solar core. The average separation has been measured with considerable precision (*e.g.* Jiménez *et al.* 1988, Gelly *et al.* 1988). However, since the asymptotic relation (3.7) is not exact, the value of D_0 depends on how it is obtained. Here I use a least squares fit (*e.g.* Scherrer *et al.* 1983) to the frequencies of modes of degree 0 - 3, including those for which $17 \leq n + \frac{1}{2}\ell \leq 29$, corresponding in frequency to the range between approximately 2500 and 4100 μHz. This leads to a determination of D_0, as well as an average value $\Delta\nu_0$ of the overall frequency spacing $\Delta\nu$ (*cf.* equation (3.6)).

I have applied this analysis to the observed frequencies, and to the frequencies compared with the observations in Figure 7. In addition I have considered a model whose core has been partially mixed; the hydrogen abundance profile $X(m)$, as a function of the mass m, was obtained from that of the $Re^* = 100$ model of Schatzman *et al.* (1981) by scaling with a constant factor, chosen to obtain the correct luminosity (*cf.* Christensen-Dalsgaard 1986). The results are shown in Table I. The most striking feature is the difference between the results for the normal and the partially mixed model. The increase in D_0 in the mixed model is caused by the increase in the sound speed c in the core. For an approximately ideal gas c is given by

$$c^2 \approx \frac{\Gamma_1 k_B T}{\mu m_u}, \qquad (5.1)$$

where T is temperature, μ is the mean molecular weight, k_B is Boltzman's constant and m_u is the atomic mass unit. In the mixed model the central hydrogen abundance is higher, and μ is consequently smaller; on the other hand there is little difference in the temperature. As a result c is higher. For the remaining models there is some scatter in the values of D_0, the general tendency being that the computed values are close to, but slightly higher than the value obtained from the observations. On the other hand D_0 for the mixed model is evidently not consistent with the observed value. Thus mixing as severe as that proposed by Schatzman *et al.* appears to be ruled out by the observed frequencies (see also Cox & Kidman 1984; Provost 1984; Christensen-Dalsgaard 1986).

It has been pointed out (Spergel, these proceedings; Faulkner 1990) that the presence in the solar core of a very small population of hypothetical "weakly interacting massive particles" (WIMPs) could contribute to the energy transport in the core and hence lower the central temperature and consequently the neutrino flux. The reduction in the central temperature, which occurs without substantial modifications in the composition profile, would lead to a reduction of the sound speed in the core, and hence to a reduction in D_0 (Faulkner, Gough & Vahia 1986, Däppen, Gilliland & Christensen-Dalsgaard 1986, Gilliland & Däppen 1988). If parameters are

Table I.

Frequencies	$\Delta\nu_0$	D_0
Duvall *et al.* observations	135.15 μHz	1.487 μHz
EFF, CT model	136.52 μHz	1.551 μHz
MHD, CT model	136.78 μHz	1.533 μHz
EFF, LAOL model	135.30 μHz	1.557 μHz
MHD, LAOL model	135.57 μHz	1.500 μHz
Schatzman *et al.* mixed model	136.68 μHz	1.974 μHz

Average frequency separations (*cf.* equations (3.5) and (3.7)) for the compilation of observed frequencies by Duvall *et al.* (1988), as well as for a number of solar models. The first four models are "standard" solar models, differing in the equation of state or the opacity, whereas in the last model the hydrogen profile simulates partial mixing by "turbulent diffusion" (Schatzman *et al.* 1981).

chosen for the WIMPs such that the model has the observed neutrino capture rate, D_0 is typically reduced by 8 - 15 per cent relative to the corresponding normal models. The earlier calculations indicated that this improved the agreement between the computed and the observed values of D_0. However, the values presented in Table I are somewhat smaller than those obtained previously for the normal models, the difference being probably due to an improvement in the numerical precision. Thus it appears likely that for corresponding models with WIMPs D_0 would be significantly *lower* than the observations. Certainly there is no evidence in the present results that modifications beyond the "standard" model is required to bring theory and observations into agreement on D_0 (see also Cox *et al.* 1989). Indeed, Gough & Kosovichev (1988) found that the sound speed inferred from inverting more extensive sets of oscillation frequencies appeared to be inconsistent with models including WIMPs (see also Gough, these proceedings).

6. Discussion.

The principal result of the present paper is that solar oscillation frequencies are sensitive to the physics of the solar interior, and that by suitable analysis of the observations it is possible to to some extent to separate the effects of various aspects of the physics. The separation is aided by the properties of the oscillations, in that the observations contain modes that penetrate to very different depths. Furthermore, the structure of the convection zone is largely independent of opacity; by studying modes that are entirely trapped in the convection zone, one therefore gets information about the equation of state that is depends little on the uncertainties in the opacities. On the other hand, beneath the convection zone the gas is essentially fully ionized and the equation of state is relatively simple; here the principal uncertainty is therefore in the opacity. Thus, by judicious choice of data there is some hope that we may learn both about the equation of state and the opacity of matter under stellar condition.

A special problem is encountered near the solar surface, where the physics of both the structure of the Sun and the oscillations is uncertain. However, by suitable scaling of frequency differences between the observations and the model the effects of this region can to a large extent be eliminated. Thus it is possible to study the properties of the solar interior separately from the uncertainties of the surface region.

These principles were illustrated by studying the effects on the model and the frequencies of artificial modifications to the opacity, and by comparing with the observations frequencies of models differing in the equation of state or the opacity. The differences in the the physics were within the range of currently used formulations. These differences caused frequency changes that far exceeded the observational errors. Furthermore it appeared that the use of a more sophisticated equation of state lead to a considerable improvement between observed and computed frequencies. The situation with regards to the opacity is less clear. The use of the newer opacities in the solar interior apparently increased the discrepancy between theory and observation; on the other hand a recently proposed major increase in the opacity in the solar atmosphere seemed to cause a significant improvement in the agreement, reducing the differences between theory and observation to a level close to the observational error for those modes that are trapped in the convection zone. Given the remaining uncertainties affecting this part of the model, however, this agreement is of doubtful significance.

I have only considered the simplest use of the observed oscillation frequencies. Much more detailed information can be obtained by applying *inverse analyses* (*e.g.* Gough 1985, 1986b; Gough & Kosovichev 1988; Gough & Thompson 1990). The data are in fact of sufficient quality to permit the determination of the run of sound speed as a function of position in much of the Sun (*e.g.* Christensen-Dalsgaard *et al.* 1985; Brodsky & Vorontsov 1987; Vorontsov 1988; Kosovichev 1988; Christensen-Dalsgaard, Gough & Thompson 1988, 1989; Sekii & Shibahashi 1989), or for other properties of the solar interior. Gough & Kosovichev (1988) and Kosovichev (1990) suggested that the data indicate slight mixing of material in the solar core, although to a far smaller extent than in the Schatzman *et al.* (1981) model discussed in section 5. Dziembowski, Pamyatnykh & Sienkiewicz (1990) found that the minimum neutrino flux consistent with results of an inversion were considerably *higher* than the flux predicted by standard solar models; the sensitivity of this conclusion to random and systematic errors in the data has still to be tested, however.

Helioseismology has already contributed significantly to our knowledge about the interior of the Sun. It is perhaps surprising (and to some possibly even disappointing) that so far no major departure from standard evolution theory has been revealed by the results. Certainly they have offered no solution to the neutrino problem; in contrast there is a tendency for models with a low neutrino flux (*e.g.* with substantial core mixing or energy transport by WIMPs) to be inconsistent with the seismic data. We are only the at the beginning of seismic investigations of the Sun, however. In the coming decade we shall witness a major increase in the amount and quality of observations of solar oscillations, as ground-based networks of oscillation observatories (Hill, these proceedings) and space-based facilities (Bonnet, these proceedings) become operational. Such data will allow us to look for more subtle failures of standard models. We can hope to constrain conditions in the solar core to the extent that a reliable estimate can be made of the neutrino spectrum which is produced by nuclear reactions; the detailed observations of the neutrino spectrum which will become available in the same period could then be used to investigate the properties of the neutrino. Finally, it should be possible to separate the uncertainty in the "macrophysics" of the solar interior from the effects of the microphysics, and hence to investigate the latter in considerable detail; we would then be in a position to use the Sun as a laboratory for the study of basic properties of plasmas.

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