independence proof for the continuum hypothesis, which shows clearly that toposes are usefully considered to be "Heyting-valued models" of set theory; and finally, there is an appendix on Penon's locally internal categories.

The book is throughout clear and precise, with an interesting historical introduction and plenty of helpful remarks and illustrations. Further developments are indicated in over a hundred exercises, and there are excellent bibliographies (usefully including references to Mathematical Reviews) and indices of definitions, notations and names. And just as the significance of the subject is as an attitude rather than as a technique, so the importance of the book is not as an assembly of original or unpublished ideas in topos theory but as a coherent account, essential reading for any topos theorist wishing to understand and master his subject, and an excellent introduction for everyone else; it is not likely to be superseded by a better work for some years. I regret the shortage, occasionally, of examples to bring the theory down to earth—for example, of object classifiers, or of coherent toposes; and of applications of results such as Barr's theorem; the inexperienced reader will probably find the exercises rather hard. But on balance I am very happy with the level of treatment: it would be hard, in a more leisurely introduction, to give such a good idea of the scope, depth, and interest of this increasingly important subject.

R. DYCKHOFF

JORDAN, D. W. and SMITH, P., *Nonlinear Ordinary Differential Equations* (Oxford Applied Mathematics and Computing Series), 360 pp, £12·00 (hard cover).

The aim of this book is to provide an undergraduate text dealing with the techniques used to obtain exact or approximate solutions of ordinary differential equations. This topic is excellently motivated, wherever possible, by elementary dynamical situations giving a physical background to the mathematical theory. The reader is then left with a clear intuitive picture of what would otherwise be a purely abstract concept.

The book starts with the conventional phase plane analysis and then spends several chapters on perturbation methods. This extensive study covers the various techniques of singular perturbation theory, averaging, forced oscillations, harmonic and subharmonic response and differential equations with periodic coefficients. The book also covers Liapunov stability and has a section on existence of periodic solutions. A large number of these topics would not be out of place in a postgraduate course. However the authors have skilfully managed to introduce everything at an elementary level so that no final year undergraduate student should feel that the underlying principles are beyond him. This does not imply that the authors deal lightly with such topics—on the contrary, one is led through quite complicated mathematical detail with expert care. The text is also backed up by very good figures and many illustrative and instructive examples, some worked and some left as exercises. These have clearly been culled from many years of searching for new questions to complement the course of lectures from which the book has been developed.

The book is mainly methods oriented, the aim being to explain the mathematics behind the various techniques involved. This it does very well. It also covers some theorems regarding existence and uniqueness which is in any case rather limited in this sphere of mathematics. It does not however deal with some of the more difficult aspects—for example, error estimation for the various approximations. Some of these aspects have been covered in great detail by other authors and are probably best left out of a book of this nature.

J. G. BYATT-SMITH

PETRICH, MARIO, *Lectures in Semigroups* (Wiley, 1977), 164 pp.

If this text were handed to me without the author's name I should have had no difficulty in guessing correctly who had written it. The style is unmistakeable. To coin a new adjective, it is truly "petrich": minimally encyclopaedic (which, alas, naturally implies the adjective "dreich"). It is most certainly not an elementary text and reading it is not an easy task, the wealth of material that it contains rendering it difficult to digest quickly. Nevertheless, it will be enjoyed by a happy, albeit small, group of readers; for a considerable amount of material of Eastern European and Russian origin is presented here in English for the first time. The first chapter is introductory and the author begins Chapter II with the notion of a band (a semigroup in which $x^2 = x$ for all x).

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The account given is by far the most comprehensive to date. In Chapter III he considers rectangular bands ($xyx = x$ for all x, y) and what he calls matrix decompositions of a semigroup (such a decomposition is induced by a homomorphism onto a rectangular band). Chapter IV is devoted to normal band decompositions, a normal band ($xyzy = xzyx$) being, in the variety of bands, the join of a semilattice ($xy = yx$) and a rectangular band. In Chapter V the main object of study is the lattice of subsemigroups of a semigroup and the obvious problems of when this lattice is modular, distributive, complemented, etc. Nowadays, with Mathematical Reviews coping with more than 2,500 papers per month, it is essential that top notch mathematicians such as Petrich should devote a lot of their time to producing survey research monographs. But, please, in the galaxy of publications let each of these not have the appearance of a white dwarf.

T. S. BLYTH

KOLMAN, B., *Introductory Linear Algebra with Applications* (Macmillan, 1976), xvi+426 pp.

This book is intended as an introduction to linear algebra, with applications (in the social sciences), and with some emphasis on numerical linear algebra. To construct such a book is a challenging task and the author has had some success. I am afraid, however, that there are many details about which I am not happy, and which, taken together, tend to diminish the usefulness of the book. Since the text falls fairly clearly into three parts I shall discuss these separately.

First, the theoretical linear algebra part is constrained to finite-dimensional vector spaces over the real field (complex numbers are strictly forbidden, even as eigenvalues of real unsymmetric matrices). An axiomatic approach is used, but several theorems are given without proof (especially concerning matrix rank). No attempt is made to draw a distinction between a vector space (only $\boldsymbol{R}^n$ is considered in any detail) and its coordinate space with respect to a basis. Indeed these concepts are identified, and vectors are notated as $n \times 1$ matrices. This use of the same notation for matrices and vectors (capital Clarendon type) makes some expressions unclear. Some abstract theory is presented, but is rather unsatisfactory; the definition of vector space using general operations requires bracketing in places, because of the finite-dimensional assumption linear transformations appear little different from matrices (and are often specified using matrices), and no definition of composition of these is given. It is a pity that the author omitted the unifying result that the general solution of a linear equation is a particular solution plus the general solution of the associated homogeneous equation. Perhaps some of these points could have been improved at the cost of much of the material on determinants, which I found to be excessive in detail, and to come rather early in the text for such a relatively unimportant topic.

Second, the applications examined are linear programming, geometry, graph theory, game theory, least squares, and economic models. None of these involves any calculus. On the whole these are successful, especially the economic models which bring out well the unifying nature of linear algebra. It is a pity that the linear programming material stops short of duality, especially since all the matrix tools are available. The matrix games section is rather spoiled by three errors in the examples and an unclear definition of "constant-sum games", which may well confuse inexperienced readers.

Third, the numerical linear algebra part covers Gaussian elimination with partial pivoting, iterative solution of linear equations, the power method, and Jacobi's eigenvalue method. It is unfortunate that the recommendation of Gauss–Jordan elimination given in Chapter 1 must stand, without comment, until Chapter 8, especially since the more efficient method can be achieved by using the row operations in a different order, or even by fewer operations. (This latter method is achieved at the cost of uniqueness of the echelon form, but this is seldom useful.) Finally, the text contains the fallacy that it is computationally efficient to find the inverse matrix when several sets of linear equations with different right-hand sides are to be solved.

The text is very clearly printed, with helpful diagrams and tables. There are copious worked examples at all levels of difficulty, with many exercises, solutions being given to half of them. I observed about a dozen misprints, mostly trivial. In addition to the errors already mentioned, the definition of congruence is later contradicted by use of a non-orthogonal transformation, and some of the discussion about degeneracy in linear programming is wrong.