1 ZODIACAL LIGHT

1.3 Models and Interpretation

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1.3.1

SOME FORMULAE TO INTERPRET ZODIACAL LIGHT PHOTOPOLARIMETRIC DATA IN THE ECLIPTIC FROM GROUND OR SPACE

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A fundamental step towards the knowledge of interplanetary matter from zodiacal light photometry is to eliminate the integral along the line of sight - an intrinsic, cumbersome feature of all z.l. observations - so as to reach, by inversion, the local optical properties of elementary volumes of space.

Let $\mathcal{J}(r, \theta)$ be the intensity (per steradian) of sunlight scattered, under the scattering angle θ, by a unit-volume of space situated at r A.U. from the sun. Let r_o be the heliocentric distance of a space probe containing a photometer, which aims in some direction whatever; the z.l. observed will be the integral

$$Z = \int \mathcal{J}\, d\ell \tag{1}$$

for the whole line of sight, where ℓ is the distance of the probe to an elementary current slice of the exploring narrow cone. Eq. (1), of course, is also valid near r_o= 1 A.U., viz. in the case of ground-based, balloon-borne or satellite-borne experiments.

SPACE DENSITY RUN IN THE ECLIPTIC AND PHASE FUNCTION (GENERAL FORMULAE)

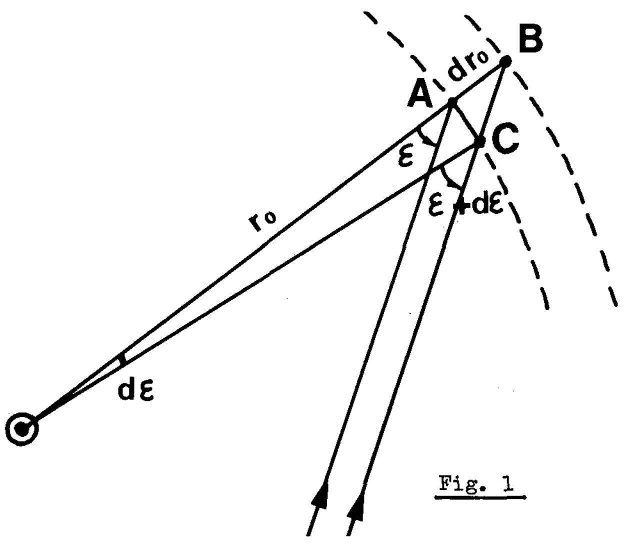

Fig. 1

Suppose the photometer to be and to aim in the symmetry plane of the zodiacal cloud. Assuming the local properties of dust in that plane to depend upon r only (rings or gaps of dust possible, but no deviation to circular symmetry around the sun), the photometer will observe the z.l. brightness Z, function of the two variables r_o and ε (elongation). Fig. 1 allows us to introduce

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elementary differences between the three neighbouring locations A,B,C assumed for the probe.

When going from A to B without change of elongation, we lose in zodiacal brightness:

$$Z(A)-Z(B) = -\left(\frac{\partial Z}{\partial r_0}\right)_{\varepsilon, r_0} \cdot AB \tag{2}$$

(the partial derivative is taken at ε = cst and r_0 variable). When going from B to C with the same line of sight, the loss will be, according to eq. (1) :

$$Z(B)-Z(C) = \mathcal{J}(r=r_0, \theta=\varepsilon) \cdot BC \tag{3}$$

When returning in A, the increase will be:

$$Z(A)-Z(C) = -\left(\frac{\partial Z}{\partial \varepsilon}\right)_{r_0, \varepsilon} \cdot d\varepsilon \tag{4}$$

If we replace in (2) AB by dr_0 ; in (3) BC by $dr_0 \sec \varepsilon$; and in (4) $d\varepsilon$ by $dr_0 \, tg\varepsilon / r_0$, the loop ABCA leads us to write:

$$\mathcal{J}(r=r_0, \theta=\varepsilon) = \cos\varepsilon \left(\frac{\partial Z}{\partial r_0}\right)_{\varepsilon, r_0} - \frac{1}{r_0} \sin\varepsilon \left(\frac{\partial Z}{\partial \varepsilon}\right)_{r_0, \varepsilon} \tag{5}$$

Let $\Phi(r)$ be the total energy scattered by a unit-volume in all directions; we may write the intensity:

$$\mathcal{J}(r=r_0, \theta=\varepsilon) = \Phi(r) \cdot \sigma(\theta) \tag{6}$$

where $\sigma(\theta)$ is the phase function, normalized to unity for the whole sphere. If the properties of interplanetary dust are assumed to be the same everywhere, then $\sigma(\theta)$ does not depend on r, so that $\Phi(r)$ is proportional to the space density $\rho(r)$ and to the solar flux. Therefore, the space density near the probe may be written, in arbitrary units:

$$\rho(r_0) = r_0^2 \cos\varepsilon \left(\frac{\partial Z}{\partial r_0}\right)_{\varepsilon, r_0} - r_0 \sin\varepsilon \left(\frac{\partial Z}{\partial \varepsilon}\right)_{r_0, \varepsilon} \tag{7}$$

It would be optimistic to expect from the available as well as from the forthcoming space probe data a complete coverage of the field $Z(r_0, \varepsilon)$; however, we see that a photometer continuously aiming at the antisun can provide the gradient $\partial Z/\partial r_0$, from which the space density (in arbitrary units) is directly derived:

$$\rho(r_0) = -r_0^2 \left(\frac{\partial Z}{\partial r_0}\right)_{180°, r_0} \tag{8}$$

On the other hand, if in eq.(5) we try to cancel the other term of the right-hand side, we obtain a determination of the intensity scattered at right angle:

$$\mathcal{J}(r_0, 90°) = -\frac{1}{r_0} \left(\frac{\partial Z}{\partial \varepsilon}\right)_{r_0, 90°} \tag{9}$$

so that, for $r_0 \cong 1$ A.U., this intensity can be derived without any space probe data, nor assumption about the heliocentric dependence of the space density.

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Since we have assumed $\sigma(\theta)$ to be the same everywhere, and especially at 1 A.U., we have, in arbitrary units:

$$\sigma(\theta=\varepsilon) = \cos\varepsilon\left(\frac{\partial Z}{\partial r_o}\right)_{\varepsilon,1} - \sin\varepsilon\left(\frac{\partial Z}{\partial \varepsilon}\right)_{1,\varepsilon} \tag{10}$$

Up to now, in order to obtain the phase function over a wide range of θ, not only earthbound observations ($r_o = 1$) of Z over the same wide range of ε seem to be required, but moreover the gradient in r_o – a quantity presently known only for a few elongations.

GRADIENT OF Z.L. WITH HELIOCENTRIC DISTANCE AND PRACTICAL FORMULA FOR THE PHASE FUNCTION

Fortunately, a simplification arises if we assume the space density ρ to follow a regular law and to be proportional to r_o^{-n}.

Consider (fig. 2) two locations of the photometer, aligned with the sun, and two parallel lines of sight. Consider a secant pivoting around the sun, and let it carve the two beams in corresponding elements denoted M' (length: dl') and M" (length: dl"). We have $dl''/dl' = 1 + (dr_o/r_o)$. Since the scattering angle is the same, eq. (6) shows that the ratio of the intensities scattered towards B and A by unit-volumes situated at M" and at M' is $\mathcal{J}''/\mathcal{J}' = \Phi(\odot M'')/\Phi(\odot M') = [1 + (dr_o/r_o)]^{-(2+n)}$. When integrating along the two beams we obtain, according to eq. (1), $Z(B)/Z(A) = (\mathcal{J}''/\mathcal{J}').(dl''/dl') = [1 + (dr_o/r_o)]^{-(1+n)} = 1 - (1+n)(dr_o/r_o)$.

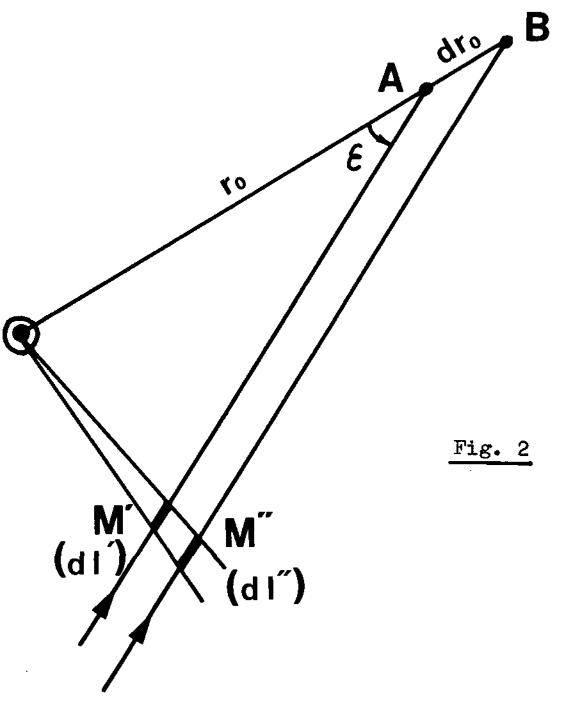

Fig. 2

Therefore,

$$\left(\frac{\partial Z}{\partial r_o}\right)_{\varepsilon,r_o} = -\frac{1+n}{r_o} Z(r_o,\varepsilon) \tag{11}$$

so that eq. (10) becomes (at 1 A.U., and still in arbitrary units):

$$\sigma(\theta=\varepsilon) = -(1+n)\cos\varepsilon\, Z(\varepsilon) - \sin\varepsilon\,\frac{dZ}{d\varepsilon} \tag{12}$$

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In the above assumption, and in so far as the parameter n can be determined (essentially with space probe data), the phase function can be derived from a photometric survey along the ecliptic at 1 A.U. In another paper (Dumont 1976) we derive the phase function from $\theta = 15°$ to the antisun, according to the $Z(\varepsilon)$ data of Leinert et al. 1974 (rocket), Frey et al. 1974 (balloon), and to the ground-based data of Haleakala (Weinberg 1964) and of Tenerife (Dumont and Sánchez 1975). We assume the most probable value of n to be 1.2.

It might be argued against the validity of eqs. (11) and (12) that a perfectly smooth law such as r^{-n}, even if fitting acceptably the true run of the space density in the inner solar system, is more and more unlikely very far from the sun when $r \to \infty$. The fact that a complete fall of zodiacal brightness, therefore of density, is reported by Pioneer 10 as crossing the asteroïdal belt, reinforces such a criticism. However, the weakness of the residual brightness when entering the belt (Hanner et al. 1974) allows to think that the lack of dust beyond 3.3 A.U. can only bring minor disturbances to the $\sigma(\theta)$ function provided by eq. (12). Moreover, if we concentrate upon the rotating secant of fig. 2, we notice that a zero-level of dust beyond a given heliocentric distance would leave the above geometrical derivation of eqs. (11) and (12) still valid (since an integration along a segment of the double-beam, instead of an infinite double-beam, would provide $Z(B)/Z(A)$ without any change to the preceding formulae).

POLARIMETRIC FORMULAE

Eqs. (1) to (6) and (9) to (12) may be written in the polarimetric fashion, i.e. separately for each Fresnel vector (1 = perpendicular to, and 2 = lying in, the scattering plane). The corresponding components of the z.l. are $Z_1(r_o, \varepsilon)$, $Z_2(r_o, \varepsilon)$, those of the intensity are $\mathcal{I}_1(r, \theta)$, $\mathcal{I}_2(r, \theta)$. The observed degree of polarization is $P = (Z_1 - Z_2)/Z$; the true local degree of polarization will be $\mathcal{P} = (\mathcal{I}_1 - \mathcal{I}_2)/\mathcal{I}$.

From a double formulation of eq. (12), and omitting here the intermediate steps (see Dumont 1973; Leinert 1975), we are led to the following expression of $\mathcal{P}$, which generally differs from P :

$$\mathcal{P}(r = r_o, \theta = \varepsilon) = P(r_o, \varepsilon) - \frac{1}{\sigma(\theta = \varepsilon)} \sin\varepsilon \, Z(r_o, \varepsilon) \left(\frac{\partial P}{\partial \varepsilon}\right)_{r_o, \varepsilon} \qquad (13)$$

where n, and its uncertainty, only appear through σ. Therefore, at $\theta = \varepsilon = 90°$, n vanishes from eq. (13) since it vanishes from eq. (12). A second value of θ ruling n out will be $\theta = \varepsilon_M$, i.e. the elongation of the maximum of observed polarization P. The existence of those two particular values of θ allowing to compute $\mathcal{P}$ independently of n involves that the whole function $\mathcal{P}(\theta)$ is rather weakly sensitive to the value of n adopted (Dumont 1976).

Our assumption that dust properties, except its space density, do not depend

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on r, implies that $\mathcal{P}$ also is independent of r (at a given constant scattering angle θ). Besides, a double formulation of eq. (11) shows that P also has to be independent of r_o as far as the density law r^{-n} is valid, because calling J the quantity Z_1-Z_2 = PZ, and omitting the indices ϵ and r_o in the derivatives, we have

$$\frac{\partial J}{\partial r_o} = -\frac{1+n}{r_o} J$$

$$\frac{\partial P}{\partial r_o} = \frac{1}{Z^2}\left[Z\frac{\partial J}{\partial r_o} - J\frac{\partial Z}{\partial r_o}\right] = \frac{1}{Z^2}\left[-\frac{1+n}{r_o} ZJ + \frac{1+n}{r_o} ZJ\right]$$

so that, at a constant elongation,

$$\frac{\partial P}{\partial r_o} = 0 \tag{14}$$

This result, compared to the forthcoming data of deep space probes, could be a test of validity for the assumptions that have been made.

In practice, eqs. (7), (8), (10) and (11) could be of interest when interpreting these space probe data; eqs. (9) and (12) are able to extract valuable informations from the observational data near 1 A.U.; eq. (13) provides the polarization curve of the scatterers, which according to eq. (14) is expected to be independent of heliocentric distance.

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