

**PART 7.**

**Local And Global Instabilities And Disk  
Perturbations**

## **Instabilities in Accretion Disks of Cataclysmic Variables: A Unification Model for Dwarf Nova Outburst**

Yoji Osaki

*Department of Astronomy, University of Tokyo, Bunkyo-ku, Tokyo, 113*

**Abstract.** Instabilities of accretion disks in cataclysmic variable stars are reviewed in relation to dwarf nova outbursts. Two different kinds of instabilities of accretion disks are now known: the thermal instability and the tidal instability. The thermal instability is produced by hydrogen ionization-recombination transition, which gives rise to a thermal limit-cycle oscillation in accretion disks and it is thought to be responsible for outbursts of U Gem-type dwarf novae. The tidal instability is produced by the tidal effects of the secondary star on accretion disks, by which the disk is deformed to eccentric form and it slowly precesses in the inertial frame of reference. The tidal instability is thought to be responsible for the superhump phenomenon observed during superoutbursts of SU UMa-type dwarf novae. There is a rich variety in outburst behaviors of non-magnetic cataclysmic variables, starting from non-outbursting nova-like stars to various sub-classes of dwarf novae. A unification model for dwarf nova outbursts is then proposed based on these two instabilities. In this model, the non-magnetic cataclysmic variables are classified in the orbital-period versus mass-transfer-rate diagram into four regions depending on different combination to these two instabilities, and their observed outburst behaviors are basically understood on this diagram.

### **1. Introduction**

Dwarf novae are eruptive variables exhibiting quasi-periodic outbursts of amplitudes of 2-6 mag, of typical recurrence time of 20-300 days. They belong to a more general class of cataclysmic variable stars (abbreviated as CVs), in which a Roche-lobe filling cool dwarf star (the secondary star) loses mass through the inner Lagrangian point and a white dwarf (the primary star) accretes it. CVs are further classified into magnetic and non-magnetic systems and dwarf novae belong to non-magnetic CVs in that accretion occurs via an accretion disk.

Non-magnetic CVs exhibit a rich variety in outburst behaviors, starting from non-eruptive nova-like stars (NL), and to three basic sub-classes of dwarf novae: (1) the U Gem-type (UG) which show the most typical dwarf-nova light-curves with quasi-periodic sequence of outburst-quietness, (2) the Z Cam-type (ZC) which show rather frequent outbursts, occasionally interrupted by a "standstill" at an intermediate level between the outburst maximum and minimum for some indefinite period, and (3) the SU UMa-types (SU) which exhibit two distinct types of outbursts, a "normal outburst" lasting for a few days and a "su-

peroutburst" lasting typically for about two weeks. Usually, one superoutburst is followed by several short normal outbursts before the next superoutburst, and this sequence is called "supercycle". The most enigmatic feature of superoutburst of SU UMa stars is an occurrence of periodic light humps with amplitude of 0.2 – 0.3 mag called "superhumps", whose period is always slightly longer than the orbital period of the binary (usually by a few percent).

The WZ Sge stars (WZ), which exhibit large-amplitude outbursts of 6-8 mag with extremely long recurrence times as long as 30 years, are now thought to be an extremely case of SU UMa stars. Another class of extreme SU UMa stars has recently been discovered, called ER UMa stars (ER) which have extremely short superoutburst cycles (supercycles) less than 50 days.

The observed orbital-period distribution of non-magnetic CVs shows a bimodal character, in which their periods are either longer than 3 hour or shorter than 2 hour and systems having periods between 2 hour and 3 hour are very rare. Thus this period range is called the "period gap". U Gem stars and Z Cam stars belong to long period systems above the gap while the SU UMa stars belong to the short period ones below the gap.

A comprehensive review of both observations and theories on CVs may be found in a monograph by Warner (1995a).

## 2. Instabilities in Accretion Disks of Cataclysmic Variables

It is now well established that outbursts of dwarf novae are caused by a sudden brightening of accretion disk due to increased accretion onto the white dwarf. To explain intermittent accretion onto the white dwarf in dwarf novae, two different models were proposed in 1970s; (1) one model was the mass transfer burst model (MTB model) advocated by Bath (1973) and (2) the other was the disk instability model (DI model) proposed first by the present author (Osaki 1974). In the MTB model the mass transfer rate from the secondary star is thought to be variable and the mass accretion rate onto the white dwarf is variable accordingly. In the DI model the mass transfer rate from the secondary star is assumed to be constant in time but the alternation of outburst/quiescence is caused by (some unknown at that time) instabilities within accretion disks; mass is stored within the disk during quiescence and when it reaches some critical amount it is suddenly accreted onto the white dwarf due to some instability in the accretion disk, which explains an outburst.

In the 1970's these two ideas fiercely competed with each other. However, a very promising instability mechanism in accretion disks was discovered around 1980. This instability was a thermal instability of accretion disks, and it was based on the bi-stable nature of accretion disks with the disk temperature around  $10^4\text{K}$  where the hydrogen changes from ionized state to neutral state, and it is called the "thermal instability" or the "thermal limit cycle instability". The thermal limit-cycle instability is best explained based on the now well known S-shaped thermal equilibrium curve of the accretion disk in which there exist two stable states in accretion disks, the hot high viscosity state and the cold low viscosity state, and the intermediate state is thermally unstable. That is, mass is stored in the disk during quiescence in low-viscosity state. When mass is sufficiently accumulated to reach the turning point of the S-shaped curve, the

thermal instability sets in and the disk jumps to the hot high-viscosity state, dumping mass onto the central white dwarf, corresponding to an outburst. The outburst ends when mass is depleted in the disk and the disk drops back to a cold state once more.

Various arguments have been presented in favor of the DI model based on this thermal instability over the MTB model, and the thermal limit-cycle mechanism (or the DI model) is now widely accepted by both theoreticians and observers, at least for explanation of the ordinary dwarf nova outbursts of U Gem type (see, e.g., a review by Cannizzo 1993).

Another intrinsic instability that operates in accretion disks, called the "tidal instability" or "tidally driven eccentric instability", was later discovered by Whitehurst (1988), which was further examined by Hirose and Osaki (1990) and by Lubow (1991) and by Murray (1996). The tidal instability is produced by the tidal effects of the secondary star on the accretion disk, by which instability the accretion disk is deformed to an eccentric form and it slowly precesses in the inertial frame of reference. Periodic tidal stressing of the eccentric disk by the orbiting secondary star produces a periodically enhanced dissipation in the accretion disk with the synodic period of the precessing eccentric disk, and the orbiting secondary. This explains the superhump phenomenon observed in SU UMa-type dwarf novae.

The tidal instability is found to occur only when the disk's outer edge reaches the 3:1 resonance radius, that is, the so-called eccentric Lindblad resonance (Hirose and Osaki 1990; Lubow 1991; Whitehurst and King 1991), thus requiring a rather large disk in units of the binary separation. This condition is found to be realized only in binary systems with extremely low mass-ratio,  $q$ , with  $q < 0.25$ , where  $q = M_2/M_1$  and  $M_1$  and  $M_2$  are masses of the mass-accreting primary star and the secondary star, respectively.

Since the mass of the secondary star in CVs is related to the orbital period,  $P_{\text{orb}}$ , by

$$M_2 \simeq (P_{\text{orb}}/9\text{hr}) M_{\odot}, \quad (1)$$

and since the typical mass of the primary white dwarf is of  $0.8 M_{\odot}$ , the condition for the tidal instability is possible exclusively in CVs below the period gap.

The present author (Osaki 1989) has proposed the so-called "thermal-tidal instability model" to explain the superoutburst phenomenon of SU UMa stars, in which the two intrinsic instabilities, the thermal instability and tidal instability, are properly coupled. This model is basically within the general frame-work of the DI model.

### 3. A Unification Model of Dwarf Nova Outbursts

An intriguing possibility has now opened in that almost all variety of outburst light curves of dwarf novae may be explained by a single paradigm: the DI model. In this unification model, different outbursting behaviors among non-magnetic cataclysmic variable systems are basically classified by two-parameters characterizing accretion disks in these systems; that is, the orbital period of the system and the mass transfer rate from the secondary star. For a given orbital period the mass transfer rate from the secondary determines the thermal stability nature of accretion disks (Smak 1983) in a sense that CV systems with high

mass transfer rate yield hot “stable” disks corresponding to nova-like systems, while those with mass transfer rate below the critical one give rise to thermally unstable disks, producing dwarf nova outbursts.

The critical mass-transfer rate is given in the disk-instability model by (see, Smak 1983)

$$\dot{M}_{\text{crit}} = \frac{8\pi}{3} \sigma T_{\text{eff,crit}}^4 \frac{R_d^3}{GM_1}, \quad (2)$$

where  $\sigma$  and  $G$  are the Stefan-Boltzmann constant and the gravitational constant, respectively,  $R_d$  is the disk radius and  $T_{\text{eff,crit}}$  is the critical effective temperature of an accretion disk below which no hot state exists. Here, we use  $\log T_{\text{eff,crit}} = 3.9 - 0.1 \log R_{d,10}$  where  $R_{d,10} = R_d/10^{10}\text{cm}$ . We then find

$$\dot{M}_{\text{crit}} \simeq 2.7 \times 10^{17} \text{g s}^{-1} (P_{\text{orb}}/4 \text{ hr})^{1.7}, \quad (3)$$

where we have assumed that  $R_d \simeq 0.35A$  and  $M_1 + M_2 \simeq 1M_{\odot}$  and  $A$  is the binary separation.

On the other hand, the orbital period of binary determines whether the tidal instability is possible or not. As mentioned before, the period gap gives approximately a dividing line between the tidally stable systems (above the gap) and tidally unstable ones (below the gap).

Figure 1 illustrates different classes of non-magnetic CVs in the  $(P_{\text{orb}}, \dot{M})$ -diagram. Various outbursting behaviors of non-magnetic CVs are understood in a unified way (an unification model) within the basic framework of the DI model in this diagram. This diagram is divided basically into four regions by different combination of stability behaviors to these two intrinsic instabilities in the accretion disks. Corresponding to these four regions different kinds of outburst behaviors are found in non-magnetic CVs.

The accretion disks in the upper right region are stable both for the thermal instability and the tidal instability and observationally we find steady accretors called nova-like systems (NL). The ordinary dwarf novae called U Gem-type (UG) are located in the lower right region where disks are thermally unstable but tidally stable. The Z Cam stars (ZC) are located in the borderline zone between these two, which show both dwarf nova outburst (thermal limit cycle oscillation) and “standstill” (steady accretion).

Moving to the upper left region in this diagram, we find another interesting class of CVs called “permanent superhumpers” (designated here as PS), which are nova-like stars (steady accretors) but nevertheless exhibit permanent superhump phenomenon (see, e.g., Skillman and Patterson 1993). These stars are understood to be permanently stuck in superoutburst because they are thermally stable but tidally unstable. The CV systems that are located in the lower left corner are in general classified SU UMa stars which exhibit superoutbursts and superhumps. These stars can be both thermally and tidally unstable. In the case of SU UMa stars there is a wide range in outburst recurrence time-scales ranging from ER UMa stars with supercycles less than 50 days, to classical SU UMa stars with supercycles of order of a year, and to the extreme of WZ Sge stars with recurrence time as long as 30 years. Their difference is basically understood by the difference in mass transfer rate in the thermal-tidal instability model as discussed in the next section.

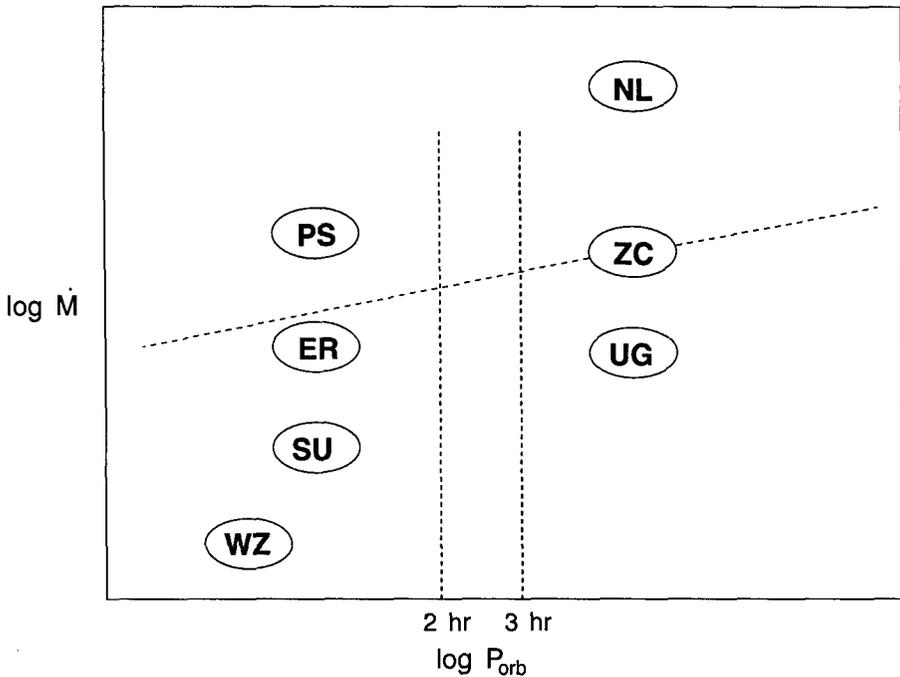


Figure 1.  $P_{\text{orb}} - \dot{M}$  diagram showing different outburst behaviors of non-magnetic CVs. The region surrounded by dotted vertical lines shows that of the CV's period gap with 2-3 hr. Dashed line shows the borderline between the thermally stable and unstable disks given by equation (3). Symbols in the figure are ; NL: nova-like stars, ZC: Z Cam stars, UG: U Gem stars, PS: "permanent superhumpers", ER: ER UMa stars, SU: SU UMa stars, and WZ: WZ Sge stars.

#### 4. Thermal-Tidal Instability Model

The present author (Osaki 1989) has proposed a model to explain the supercycle of the SU UMa stars based on the basic framework of the disk instability model. This model uses the two intrinsic instabilities of an accretion disk and it is thus called the thermal-tidal instability model.

In this model, both the normal outburst and superoutburst are caused by the thermal instability in the accretion disk. During the early phase of the supercycle, the disk is compact and the thermal instability produces quasi-periodic episodes of accretion, which are observed as normal outbursts but the accreted mass in each normal outburst is less than that transferred during quiescence because of inefficient tidal removal of angular momentum from the disk. Both the mass and angular momentum of the disk are gradually built up. The disk radius expands further with each successive outburst until it eventually exceeds the critical radius for the 3:1 resonance; this final normal outburst triggers the tidal instability, producing a precessing eccentric disk (observed as "superhumps").

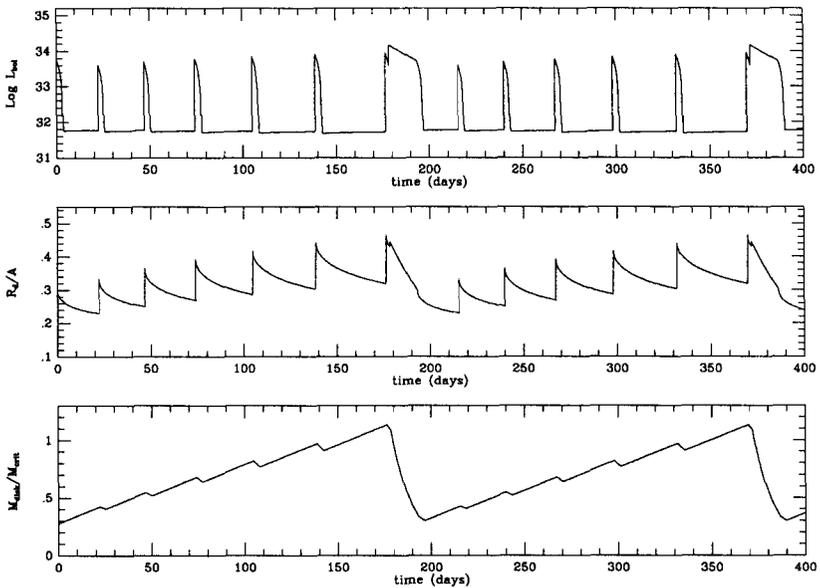


Figure 2. A numerical simulation of supercycle of an SU UMa-type star based on the thermal-tidal instability model (Osaki 1989). From top to bottom, (a) the bolometric light curve, (b) the disk radius,  $R_d$  in units of the binary separation,  $A$ , and (c) the total disk mass normalized by the critical mass above which the disk is tidally unstable.

The resulting outburst greatly clears the disk mass (producing “superoutburst”) because of greatly enhanced tidal torques due to the eccentric disk. After the end of the superoutburst, the disk returns to the starting compact state. This is the basic idea of the thermal-tidal instability model for SU UMa stars. Figure 2 illustrates results of light-curve simulations based on a simplified model developed by Osaki (1989). The supercycle of an SU UMa star is understood in this model as a relaxation oscillation cycle of the disk radius.

The most important aspect of this model is that it makes a certain prediction which is observationally testable. It concerns the variation of the disk radius throughout the supercycle as shown in the middle panel of figure 2. That is, the disk is compact in the early phase of the supercycle but it expands with an advance of the supercycle phase until it finally reaches the critical radius for the 3:1 resonance at the last normal outburst which triggers a superoutburst, while a saw-tooth-like variation in disk radius in every normal outburst is superimposed on this long-time-scale trend. Robinson et al (1995) have observed eclipsing SU UMa star called Z Cha in ultraviolet by using the Hubble space telescope, and they have found that the disk radius is slightly larger before the second normal outburst in a supercycle than the first one, which is in good agreement with the prediction of the thermal-tidal instability model.

Observations show a wide variety in activity within SU UMa stars. In fact, Vogt (1993) has classified SU UMa stars into three groups by their activity: group A, “active” stars showing frequent outbursts (e.g., VW Hyi), group B, “intermediate” in activity between two extremes (e.g., OY Car), and group C, very inactive stars or “WZ Sge” stars that exhibit large outbursts in a long interval. We may add to this activity sequence two more groups: permanent superhumpers and ER UMa stars, and these two new groups should be put above Vogt’s group A. Warner (1995b) summarizes the same observational property of SU UMa stars in a diagram exhibiting relationship between the recurrence time of normal outburst  $T_N$  and the superoutburst recurrence time  $T_S$ . We try to explain these activity sequence basically as a sequence of decreasing mass transfer rate.

#### 4.1. The activity sequence from ordinary SU UMa stars to WZ Sge stars with $\dot{M} < 10^{16} \text{ g s}^{-1}$

We discuss first the case of low mass transfer rates with  $\dot{M} < 10^{16} \text{ g s}^{-1}$  which applies to the classical SU UMa stars and WZ Sge stars. Problems of ER UMa stars and permanent superhumpers will be discussed in the next subsection.

The present author has already presented in Garching conference (Osaki 1994) and in Padova conference (Osaki 1995d) that Vogt’s (1993) activity sequence may be explained as a sequence of decreasing mass transfer rate in the thermal-tidal instability model. In this model, the supercycle length,  $T_S$ , is given for appropriate binary parameters by

$$T_S \simeq 120 \text{ day}/\dot{M}_{16}, \quad (4)$$

while the recurrence time,  $T_N$ , of normal outbursts is given by

$$T_N \simeq 14 \text{ day}/\dot{M}_{16}^2, \quad (5)$$

where  $\dot{M}_{16} = \dot{M}/10^{16} \text{ g s}^{-1}$ . Thus if the mass transfer rate is decreased along this sequence, the supercycle length is increased in proportion to the inverse of mass transfer rate while the number of normal outbursts in a supercycle decreases in proportion to the inverse of the supercycle length, in a good agreement of observed activity sequence in SU UMa stars.

However, in order to explain an extremely long recurrence time of WZ Sge itself of 30 years, it is not enough to decrease the mass transfer rate but also we need to decrease the viscosity parameter in the cold state,  $\alpha_{\text{cold}}$ . The reason for this is that the so-called inside-out type outburst always occurs within the viscous diffusion time irrespective to mass transfer rate and the viscous diffusion time is determined by the viscosity in the cold state or the viscosity parameter  $\alpha_{\text{cold}}$ . In order to avoid an earlier occurrence of normal outburst in WZ Sge, we must choose an extremely low  $\alpha_{\text{cold}} \leq 0.001$ , a conclusion reached by Smak (1993), by Osaki (1994, 1995a) and by Howell et al. (1995).

#### 4.2. High-mass transfer systems: ER UMa stars and permanent superhumpers

Let us now discuss the case with high mass transfer rate. As already noted, there is a critical mass transfer rate above which an accretion disk is always in

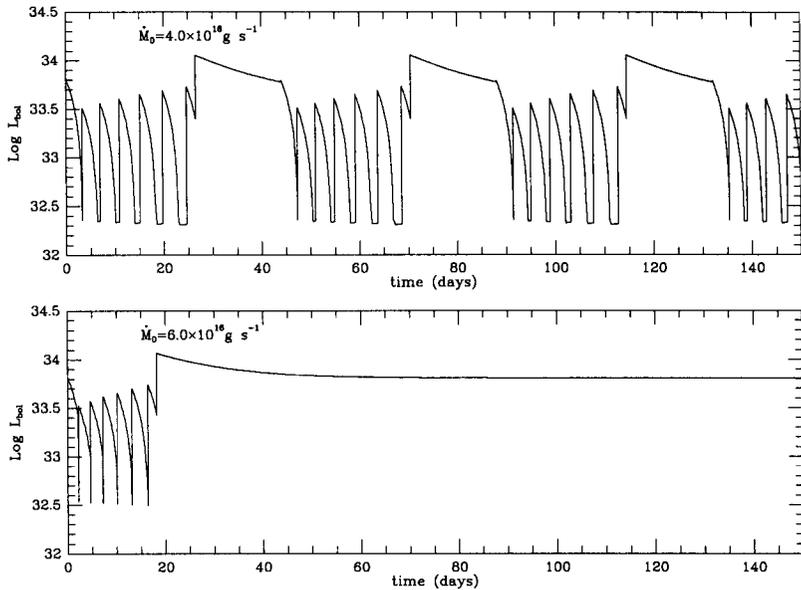


Figure 3. Light curve simulations for ER UMa star (upper panel) and a permanent superhumper (lower panel). The mass transfer rates used are, respectively,  $\dot{M} = 4 \times 10^{16} \text{ g s}^{-1}$  for ER UMa star and  $\dot{M} = 6 \times 10^{16} \text{ g s}^{-1}$  for a permanent superhumper which is just above the critical mass transfer rate.

a hot and thermally stable state. If the mass transfer rate is higher than the critical one, the system remains in a hot state. These CVs below the period gap are identified as permanent superhumpers, nova-like stars exhibiting permanent superhumps, because they are thermally stable but tidally unstable.

Osaki (1995b,c) has studied outburst light-curves of the ER UMa stars. He has examined the supercycle length as a function of the mass transfer rate under fixed model parameters. It is found that the supercycle length is inversely proportional to the mass transfer rate if  $\dot{M}_{16} < 1$  as expected. However, the mass transfer rate is further increased above  $\dot{M}_{16} \sim 1$ , the supercycle length takes a broad minimum, and it finally goes to infinity when the mass transfer rate becomes above the critical value for the thermal instability.

The upper panel of Figure 3 shows a simulated light curve for ER UMa with appropriate binary parameters, and with a mass transfer rate of  $\dot{M}_{16} = 4$  (Osaki 1995b). It is found that the simulated light-curve reproduces the observed light curve of ER UMa very well. It should, however, be noted that the mass transfer rates needed to simulate ER UMa stars are about ten times larger than those expected from the standard evolutionary scenario based on the angular momentum loss by gravitational-wave radiation (Kolb 1993). If we increase the mass transfer rate still more to that above the critical mass transfer rate, we find the permanent superhumper as shown by the lower panel of figure 3.

For model parameters used to simulate ER UMa, the minimum supercycle length was about 45 days and thus the supercycle length as short as 20 days observed for RZ LMi could not be reproduced by these model parameters. It was found (Osaki 1995c) that the minimum supercycle length is sensitive to one model parameter that describes the strength of tidal torques during the supermaximum when the disk becomes eccentric. It has turned out that the supercycle length as short as 19 days of RZ LMi can be reproduced if the tidal torques during the superoutburst in RZ LMi are significantly weaker than those of ER UMa and ordinary SU UMa stars.

## 5. Conclusion

- (1) A unification model for outburst behaviours of non-magnetic cataclysmic variable stars is proposed based on the disk instability model in which CVs are classified in the  $(P_{\text{orb}}, \dot{M})$  diagram into four regions.
- (2) Outburst behaviours of CVs below the period gap, in particular, are understood by the thermal-tidal instability model in which coupling of the two intrinsic instabilities (thermal and tidal instabilities) plays a unique role. Activity sequence of cataclysmic variables below the period gap, starting from permanent superhumpers, to ER UMa stars, to different activity classes of SU UMa stars, and finally to WZ Sge stars, may be understood as a sequence of decreasing mass transfer rates in this model.
- (3) Mass transfer rates from the secondary stars in CVs below the period gap should have much wider range than that expected from the standard CV evolutionary scenario based on gravitational-wave radiation. A cyclic variation in mass transfer rate in a time scale of  $10^3 \sim 10^4$  years is thus inferred.

A more detailed account of this model has been published as an invited review paper in PASP (Osaki 1996).

## References

- Bath, G.T. 1973, *Nature Phys. Sci.*, 246, 84
- Cannizzo J. K. 1993, in *Accretion Disks in Compact Stellar Systems*, J. C. Wheeler, Singapore: World Scientific Publishing, 6
- Hirose M., & Osaki Y. 1990, *PASJ*, 42, 135
- Howell S. B., Szkody P., & Cannizzo J. K. 1995, *ApJ*, 439, 337
- Kolb, U. 1993, *A&A*, 271, 149
- Lubow S. H. 1991, *ApJ*, 381, 259
- Murray, J. R. 1996, *MNRAS*, 279, 402
- Osaki Y. 1974, *PASJ*, 26, 429
- Osaki Y. 1989, *PASJ*, 41, 1005
- Osaki, Y. 1994, in *Theory of Accretion Disks-2*, W. Duschl, et al., Dordrecht: Kluwer Academic Publishers, 93
- Osaki Y. 1995a, *PASJ*, 47, 47
- Osaki Y. 1995b, *PASJ*, 47, L11

- Osaki Y. 1995c, PASJ, 47, L25
- Osaki Y. 1995d, in Cataclysmic Variables, A. Bianchini, M. Della Valle, & M. Orio, Dordrecht: Kluwer Academic Publishers, 307
- Osaki Y. 1996, PASP, 108, 39
- Robinson, E. L., Wood, J. H., Bless, R. C., Clemens, J. C., Dollan, J. F., Elliot, J. L., Nelson, M. J., Percival, J. W., Taylor, M. J., van Citters, G. W., & Zhang, E., 1995, ApJ, 443, 295
- Skillman D. R., & Patterson J. 1993, ApJ, 417, 298
- Smak, J. 1983, ApJ, 272, 234
- Smak, J. 1993, Acta Astron., 43, 101
- Vogt, N. 1993, in 2nd Technion-Haifa Conference on Cataclysmic Variables and Related Physics, O. Regev & G. Shaviv, Jerusalem: The Israel Physical Society, 63
- Warner, B. 1995a, Cataclysmic Variable Stars, Cambridge: Cambridge University Press
- Warner, B. 1995b, Ap&SS, 226, 187
- Whitehurst, R. 1988, MNRAS, 232, 35
- Whitehurst, R. & King, A. 1991, MNRAS, 249, 25

## Discussion

*J. Smak:* How would including the effect of enhanced mass-transfer (due to irradiation of the secondary) modify the outcome of your thermal & tidal instability models?

*Y. Osaki:* In general, enhanced mass-transfer due to irradiation of the secondary works against the tidal instability because it causes a shrinkage of the disk radius from the 3:1 resonance, and thus causing the delay of super humps. However, enhanced mass-transfer might have occurred in the 1978 super-outburst of WZ Sge, as the superhumps were delayed by ten days after the outburst maximum.