LETTER TO THE EDITOR

Dear Editor,

Pfeifer (1982) has given a proof of the following interesting fact: if $\{\tau_n\}_1^\infty$ are points of a simple point process on $R_+ = [0, \infty)$ with counting random function $N(t)$, then if $\{N(t);\ t \geqq 0\}$ is a Markov process, $\{\tau_n\}_1^\infty$ is also a Markov process. In addition, $p_{ii}(s, t) = P\{\tau_i > t \mid \tau_{i-1} = s\}$, where $p_{ij}(s, t)$ are transition probabilities of $N(t)$. In connection with these, I would like to point out that there exist more general results on this topic, some of which have already been published (Todorovic (1976)). A short proof exists, based on the following equation:

$$\mathscr{L}_n \cap \{N(t) = n\} = \mathscr{F}_t \cap \{N(t) = n\},$$

where $\mathscr{L}_n = \mathscr{G}\{\tau_1, \cdots, \tau_n\}$ and $\mathscr{F}_t = \mathscr{G}\{N(s); s \leqq t\}$, that if $N(t)$ is a Markov process, not only is $\{\tau_n\}_1^\infty$ Markov, but it has the strong Markov property. In other words, for any stopping time T with respect to the filtration $\{\mathscr{L}_n\,; n \geqq 1\}$, we have:

$$P\{\tau_{T+1} \leqq t \mid \mathscr{L}_T\} = P\{\tau_{T+1} \leqq t \mid \tau_T\} \quad \text{(a.e.)}.$$

In addition,

$$p_{j,k}(s, t) = P\{N(t) = k \mid \tau_j = s\}.$$

Both Dr Pfeifer, and the readers of the *Journal of Applied Probability*, will, I hope, wish to be informed of these results.

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Yours sincerely,
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References

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