

#### ARTICLE

# Fragmentation and the Preface Paradox

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#### Abstract

The preface paradox is often taken to show that beliefs can be individually rational but jointly inconsistent. However, this received conflict between rationality and consistency is unfounded. This paper seeks to show that no rational beliefs are actually inconsistent in the preface paradox

**Keywords:** Fragmentation; the preface paradox; belief system; consistency

## 1. The received conflict

Anna has written a history book. In it, she makes numerous assertions, each of which she believes to be true after years of study. However, recognizing her fallibility as a human, she adds the following sentence to the preface: "There is at least one false claim in this book." If she also believes this claim, which we will refer to as "the preface claim," it appears to render her belief set inconsistent. To see this, the received conflict can be derived from the following two premises:<sup>2</sup>

- The Rationality Thesis: It is rational for Anna to accept each of her individual beliefs.<sup>3</sup>
- The Consistency Principle: An agent should have a belief system (i.e., the complete set of all her beliefs) that is consistent.

The Rationality Thesis is the datum of the preface paradox. We have stipulated that Anna has good reason to believe each assertion in the main text of the book as well as the preface claim.<sup>4</sup> The Consistency Principle is a widely (but not universally) accepted epistemic principle, demanding that all agents should only accept beliefs that are consistent.5

<sup>&</sup>lt;sup>1</sup>For the original version of the preface paradox, see Makinson (1965).

<sup>&</sup>lt;sup>2</sup>Usually, the preface paradox would involve the Conjunction Principle. However, this principle is unnecessary when the focus is on the consistency of one's belief system. It will be needed when the focus shifts to the consistency of each proposition believed.

<sup>&</sup>lt;sup>3</sup>I will use the word "belief" to refer to the proposition that is believed, and not the mental state.

<sup>&</sup>lt;sup>4</sup>Note that not everyone assumes the Rationality Thesis as a given. Nelkin (2000), Ryan (1991, 1996), DeRose (1996) argue that none of the individual beliefs in the assertions is fully justified.

<sup>&</sup>lt;sup>5</sup>See Fitelson and Easwaran (2015) for an alternative epistemic requirement for rational belief.

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Note that there seems to be a conflict between the Rationality Thesis and the Consistency Principle. If Anna accepts each of her individual beliefs, then her belief system appears to be inconsistent. Either some assertion in the main text is false or the preface claim is. If she genuinely believes everything that is stated in the book, then at least one belief in her belief system is guaranteed to be false, rendering her belief system inconsistent.

However, contrary to initial appearances, I argue that the Rationality Thesis and the Consistency Principle do not conflict. To see this, let us ask: is Anna an actual human thinker or a logically omniscient agent? By "actual human thinker," I mean people like us, with limited computational powers, cognitive limitations, and fallibility. By "logically omniscient agent," I mean agents with no time or memory constraints, unlimited computational power, and perfect information access. I will then argue that if she is the former, the Consistency Principle would not be normative for her. However, if she is the latter, she would not break the Consistency Principle. I will develop arguments to support these two claims. In either case, her belief system remains consistent. This shows that no conflict arises between the premises in the preface paradox. So, my conclusion is that the preface paradox fails to illustrate that beliefs are individually rational but jointly inconsistent.

# 2. The Consistency Principle

According to the Consistency Principle, an agent should have a *belief system* that is *consistent*. Note that the principle involves two crucial terms "belief system" and "consistency" that require some clarification.

First, what is a belief system? Following Makinson (1965) and the many authors influenced by him, the Consistency Principle assumes that an agent's belief system is a "unified" whole. This applies to both ideal and ordinary agents. According to this Unified Picture, an ordinary agent's beliefs are represented as operating under a *single* unified belief system that guides all her goal-directed behavior at any given time. This view goes back to Ramsey's (1931) description of "belief as the map by which we steer." To effectively steer one's thoughts and actions, the map as a whole must be consistent. (I will say more about the Unified Picture later, but for now, the goal is to bring this assumption to our awareness.)

Second, what makes a belief system consistent? To say that one's belief system as a whole is consistent means that each of its components must be consistent with all of the others. These components are often seen as (full or partial) beliefs in the Unified Picture. In the context of the preface paradox, the Consistency Principle would amount to the requirement that rational agents accept only full beliefs that are consistent with all their other full beliefs. Let us label this second assumption as the Structural Requirement. (Note that this requirement may take on a different form in a different picture of belief. We will later introduce a non-unified picture where the components go beyond mere beliefs.)

Put together, the Consistency Principle follows from the conjunction of two assumptions: the Unified Picture and the Structural Requirement.

Some scholars have sought to refute the Consistency Principle by denying the Structural Requirement. For example, Klein (1985) argues that it can be rational to believe each individual proposition in an inconsistent set when the set includes a substantial number of propositions. He suggests that having such a belief set is epistemically

<sup>&</sup>lt;sup>6</sup>Makinson does not explicitly state the Unified Picture, but it seems to me that he is assuming it, because he says that one's belief system is just a "single" set of beliefs (1965, 206).

praiseworthy since it indicates an open mind to new disconfirming evidence and readiness for active reevaluation of existing evidence.<sup>7</sup>

In Section 4, I will show that one can accept the Structural Requirement while rejecting the Unified Picture. This approach preserves the core idea of the Consistency Principle while exposing the problem with the Unified Picture. However, first of all, I must outline the Unified Picture.

## 3. Unification and fragmentation

The Unified Picture plays a significant role in formal tools like epistemic logic, Bayesian epistemology, and decision theory. Consider, for example, the possible worlds model of belief (Hintikka 1962; Lewis 1979; Stalnaker 1984; Braddon-Mitchell and Jackson 2007) in which an agent's beliefs are represented by a set of possible worlds. The role of this set of worlds is to create a model of how the agent sees the world by identifying what they believe is possible and what they don't. The set of worlds is then used to explain the agent's goal-directed behavior. Since all beliefs are related to this one set of worlds, the resulting picture of belief is unified.

Similarly, Bayesian epistemology represents an agent's beliefs by a probability function that assigns a probability to a proposition. This probability function represents the agent's credence in the truth of the proposition. To ensure probabilistic coherence, the probability function must satisfy certain axioms of probability theory. In decision theory, an agent's preferences are also modeled by a utility function. This utility function represents the agent's subjective evaluation of the desirability of each possible outcome. To ensure coherence in preferences and choices, the utility function must adhere to certain structural axioms, such as completeness, transitivity, independence, and continuity. Since all beliefs are linked to this one probability/utility function, the resulting picture of belief is also unified.<sup>8</sup>

All these formal tools for modeling beliefs share a common feature: belief has a unified character. That is, the individual beliefs of an agent are represented as operating under a *single* unified belief system (e.g., a set of possible worlds, a probability function, or a utility function)<sup>9</sup> that guides all the agent's goal-directed behavior at any given time.<sup>10</sup> This Unified Picture aligns with Ramsey's metaphor of belief as a "map by which we steer." Although this picture may serve as a useful formal tool to model belief, it has been objected that it is based on an excessive idealization of human cognition and behavior, which rarely applies in real-world situations.<sup>11</sup>

To see why, consider the following situations.

Case 1: If you were to ask, "What was the full name of Matt LeBlanc's character in *Friends*?", I may mutter "Joey ... " and struggle to recall his last name. But if asked instead, "Was 'Joey Tribbiani' the full name of Matt LeBlanc's character in *Friends*?", I would confidently reply, "Yes, of course." Why did I struggle to respond to the first question, but not to the second?

<sup>&</sup>lt;sup>7</sup>However, see Ryan (1996) and Kaplan (1981) for a defense of the Structural Requirement.

<sup>&</sup>lt;sup>8</sup>See Elga and Rayo (2022) for the recent development of a fragmented decision theory, wherein a single state of mind is represented by a collection of credence functions, each associated with a specific choice condition.

<sup>&</sup>lt;sup>9</sup>Sometimes, a unified model may involve both probability and utility functions, such as Savage's (1954) representation theorem.

<sup>&</sup>lt;sup>10</sup>The Unified Picture usually comes with consistency and closure as normative constraints.

<sup>&</sup>lt;sup>11</sup>See Kindermann and Onofri (2021) for a detailed survey of the Unified Picture.

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Case 2: During lunch break, an email arrives informing us that afternoon classes are canceled due to severe weather. I think to myself, "That's great! I've got a free afternoon, so I'll go to the gym." Without hesitation, I make my way to the gym. However, when I finally arrive, I immediately realize that the gym is also closed due to severe weather. I turn around and talk to myself, "Of course, it's closed." My reaction when I got there – slapping my forehead and feeling embarrassed – strongly suggests that deep down, I believed, or even knew, that it would be closed. It seems inappropriate to attribute ignorance to my action. However, why would I still go to the gym if I had already believed that it would be closed?<sup>12</sup>

These cases are difficult to explain within the Unified Picture. In a unified belief system, one has constant access to all their beliefs and can incorporate them into their decision-making process. There is no place for variable access to beliefs or for failure to incorporate beliefs. If an ordinary agent's belief system is indeed a unified whole, why are phenomena like these so common? It raises doubts about the accuracy of the Unified Picture.

To explain these two cases, we can appeal to the fragmented picture (Lewis 1982, 463; Stalnaker 1984, Chapter 5). In this picture, an ordinary agent has multiple fragmented or compartmentalized belief systems (i.e., fragments), each of which guides her goal-directed behavior in different contexts. Instead of representing an ordinary agent's beliefs by a single set of propositions she believes, the fragmentationist represents beliefs as *a collection of sets of propositions* she believes. These sets, known as "fragments," serve as guides for her actions or deliberations in different contexts. The total belief system of an ordinary agent contains multiple fragmented compartments, each represented by whatever the Unified Picture uses to represent the totality of the agent's belief system.<sup>13</sup>

The fragmentationist provides natural explanations for both cases. In the first case, our ability to recall beliefs varies depending on the situation. While my belief about Matt LeBlanc's character name is stored somewhere in my mind all the time, I can only access it for the purpose of answering the second question, but not for the first. This illustrates Yalcin's (2018) proposal of "belief as question-sensitive."

In the second case, we sometimes fail to incorporate beliefs into our actions or decisions. After reading the email, I should have believed that classes were canceled and that the gym would also be closed if classes were canceled. Despite having these beliefs, I focused only on the class cancelation and overlooked its impact on the gym. This illustrates how my beliefs remained compartmentalized and did not work together in my decision-making.

In addition to explanatory power, fragmentation is also cognitively plausible. Numerous studies on human memory reveal that our "mental hard disk" of long-term memory is too large for us to perform exhaustive searches for information to retrieve and utilize in working memory. Long-term memory operates in a partitioned manner, storing information in fragments, which can lead to the oversight of interconnections between claims across different sectors (Klatzky 1975). This explains why not all of our beliefs are active at any point and why not every belief influences every decision.

Fragmentation also plays various roles in epistemology. Some authors (e.g., Kindermann and Onofri 2021) see some distinctions in attitude as a kind of fragmentation. According to this perspective, Stalnaker's (1984) belief/acceptance distinction, Schwitzgebel's (2010, 2001) distinction between belief and in between

<sup>&</sup>lt;sup>12</sup>This type of scenario was first introduced in Egan (2008), (2021) to highlight the difference between ignorance and failure to integrate beliefs.

<sup>&</sup>lt;sup>13</sup>While my primary focus is on belief, I do not dismiss the possibility that the mind (e.g., mental states like knowledge or desire) can also be fragmented.

believing, Gendler's (2008a, 2008b) belief/alief distinction, Fleisher's (2021) belief/endorsement distinction, and Greco's (2014) implicit/explicit belief distinction, among others, might better be treated as instances of fragmentation.

I hope I have raised some concerns about the Unified Picture. If ordinary agents' belief systems are truly unified, phenomena in the cases above could be very hard to explain. This problem suggests that the Unified Picture may not accurately describe how ordinary agents' belief systems actually work.

## 4. Anna's belief system isn't unified

With the necessary knowledge for the Unified Picture, we can now ask this question: Is Anna's belief system unified? I will now argue that, regardless of how we answer this question – yes or no – we ultimately reach the same conclusion: her belief system is consistent. If this is true, no conflict will arise between the Rationality Thesis and the Consistency Principle. Therefore, the preface paradox fails to establish the worrying conclusion that beliefs are individually rational but jointly inconsistent.

Suppose that Anna's belief system is not unified. This means that the Unified Picture is false because it claims that all ordinary agents have unified belief systems. Recall that the Consistency Principle follows from the conjunction of two assumptions: the Unified Picture and the Structural Requirement. If the Unified Picture is false, then the Consistency Principle is based on a false assumption and does not apply to ordinary agents. Since the principle does not apply to Anna, we have no reason to call her beliefs inconsistent. So, her belief system remains consistent.

Are there any good reasons for thinking that Anna's belief system is not unified? Yes, let me walk you through the process here. Recall that Anna rationally believes all the assertions she makes in her book, which we will call  $p_1, p_2, \ldots, p_n$ , such as

 $p_1$  = World War One is also called the Great War.

 $p_2$  = The Berlin Wall collapsed in 1989.

 $p_3$  = The United States dropped two atomic bombs in Japan.

 $p_4$  = Neil Armstrong and Edwin Aldrin walked on the Moon.

And so on.

Imagine that if I were to ask her "Is World War One also called the Great War?", she would respond with "Yes." And if I were to ask "Did the Berlin Wall collapse in 1989?", she would also respond with "Yes." In general, Anna can provide correct answers to these questions because she has the relevant information in her memory in advance that is sufficient to answer the questions. If I were to continue asking her questions based on her assertions in the book, her answers to each question would consistently be "Yes." After reviewing the entire book, if I were to ask the final question "Do you believe that there is at least one false assertion in your book?", then if Anna is a modest person, her response would be "Yes." She acknowledges this point in the preface of her book.

However, here is the issue: if Anna's belief system were a unified whole, why did she answer "Yes" instead of "No" to the final question? That is, why did she include an additional sentence in the preface acknowledging her fallibility? This would be very difficult to explain if Anna's beliefs were unified. Similar to the cases from the previous section, the more attractive conclusion, it seems to me, is that Anna's belief system is *not* unified. We can summarize the reasoning as follows:

- (P1) If Anna's belief system is a unified whole, then she is not justified in believing the preface claim.
  - (P2) But she is justified in believing the preface claim.
  - (C) Therefore, her belief system is not a unified whole.

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(P1) is an application of the Unified Picture's definition, which posits that beliefs are "always on," i.e., constantly active and retrievable. <sup>14</sup> We need not accept or deny this assumption. What matters is that for an ordinary agent with a unified belief system, information in her memory is either encoded in the single system used to represent her belief states or it is not. There is no room for variable access to information. Therefore, if Anna had all the relevant information in her memory to answer the previous questions, she should have answered "No" to the final question. In other words, if her belief system were unified, she would not be justified in believing the preface claim.

Note also that if Anna had a single unified belief system with perfect access to her beliefs, she should have easily incorporated them into her response to the final question. The assumption here is that Anna is now like a logically omniscient agent, with no time or memory constraints, unlimited cognitive power, and a unified belief system. So, her affirmative answers to the previous questions should logically have led her to answer "No" to the final question, as she would have had no grounds for doubt.

Compare: imagine that I have written a book with only two assertions. If I vividly recall both these assertions and their justifications, it seems absurd for me to add the preface claim to my book. Likewise, if Anna vividly recall all her assertions and their justifications, it seems absurd for her to add the preface claim to her book. The difference

<sup>&</sup>lt;sup>14</sup>Quilty-Dunn and Mandelbaum (2018), Bendaña (2024) note that, despite seemingly metaphysically innocuous, unification or fragmentation has some bearing on the nature of belief. For example, fragmentation supports representationalism, which holds that belief involves a specific relation to a mental representation (Fodor 1978; Field 1978; Burge 2010; Quilty-Dunn and Mandelbaum 2018). Fragmentation allows representationalists to distinguish between active and inactive, yet equally real, beliefs. In simple terms, active beliefs are mental representations stored within one's activated fragment(s), fulfilling the functional roles of belief (e.g., action guidance, deliberation, and reasoning). Inactive beliefs are mental representations stored in one's inactivated fragment(s), with only limited access by cognitive processes. So, the fragmented representationalists, who claim that even if one has a single web of belief, not all beliefs are always active, would likely reject the assumption that beliefs are "always on." See Bendaña and Mandelbaum (2021) for an argument supporting fragmentation based on evidence from cognitive science and psychology. In the metaphysics of belief, dispositionalism is the main rival account. It holds that belief involves nothing but a certain set of dispositions (Ryle 1949; Stalnaker 1984; Schwitzgebel 2002). The debate between the representationalist and the dispositionalist continues to be a lively topic. In a recent paper, Quilty-Dunn and Mandelbaum (2018) highlight several shortcomings of dispositionalism, and these shortcomings all point to one major problem: dispositionalism cannot adequately explain complex psychological phenomena like sorting tasks, cognitive dissonance, belief opacity, and belief fragmentation. Take fragmentation as an example, as it is the core of this paper. As an anti-realist about cognitive architecture, dispositionalism does not seem well-suited to explaining belief fragmentation, where inconsistent beliefs coexist and lead to different behaviors in different contexts. In contrast, representationalism sits well with the fragmented picture of belief. According to this view, beliefs are representational states stored in the mind, with separate areas for different sets of beliefs. Contradictions can persist in a single person's beliefs because inconsistent sets of beliefs are stored separately. For instance, see Mandelbaum (2015) for his fragmented representationalist explanation for the divergence between implicit and explicit attitudes. Bendaña (2024) extends this fragmented representationalist view to address the undergeneration problem, implicit attitudes, and the rules of rationality. For an opposing view, in addition to the sources cited above, see Schwitzgebel's series of papers (2001), (2002), (2010), (2013), (2023), where he defends dispositionalism and presents several arguments to show that representationalism overcommits to an unrealistic and unnecessary cognitive architecture. This short paper cannot fully explore the intricate dispute between representationalism and dispositionalism. Nor does it have enough space to present the detailed arguments on both sides. But I do think the link between fragmentation and the metaphysics of belief is crucial and deserves our attention. I hope this brief footnote serves as a side note for those interested in exploring these issues further. For a deeper engagement with the topic, the cited works provide a more thorough examination of both positions.

between Anna and me is that she is a logically omniscient agent, while I am not. So, I can only handle a two-sentence book, whereas she can handle a book of regular length.

Perhaps, one might respond to what I said by arguing that even if Anna's belief system is unified, she is still justified in believing the preface claim. There might be evidence against one of her assertions in the book that she had never encountered. However, this line of response seems to lead to a form of skepticism about justification (Foley 1987), which results in the unacceptable consequence that we cannot know or even rationally believe any empirical facts.

Consider this example: my everyday experience makes it rational for me to believe that the room I just left still has furniture in it.<sup>15</sup> We usually assume that we know and rationally believe such empirical facts. However, it is possible that the furniture is gone because a highly skilled thief has just taken it, that aliens from other universes have removed it, or that Descartes's evil demon is deceiving me, among other possibilities. If we must take into account these remote possibilities, we would not be justified in believing that the room still has furniture. If this is true, we cannot know or rationally believe most, if not all, empirical facts. Such skepticism would make it nearly impossible to plan for the future. Therefore, I find this line of response unconvincing.<sup>16</sup>

These considerations provide reasons for accepting (P1): that the Unified Picture implies a *negative* answer to the final question. In other words, if Anna has a unified belief system, such an idealized agent has no reason to believe the preface claim.

Now consider (P2). Anna, like any ordinary agent, has good reason to believe the preface claim. It is a truism that ordinary agents like us are finite creatures, with limited computational power and memory access. No real person can have an excellent memory to recall every assertion in a book, hold the complete content in their consciousness, and incorporate them into their actions or decisions. <sup>17</sup> Anna, being an ordinary agent, is also subject to these same cognitive limitations.

Suppose for the sake of illustration that Anna can overcome these limitations. Is it now reasonable for her to disbelieve the preface claim? Note that for her to justifiably disbelieve it, she must not only know that she correctly recalls the complete content of her book, but must also continue to believe them rationally. And for her to continue to believe them rationally, she must either have the original justification for each of the assertions or she must have some other justification that is sufficient for her to hold the assertions rationally. Thus, it is not enough for her to merely hold the complete content of her book in mind; she must also have sufficient justifications for each assertion simultaneously to maintain her rational acceptance of them (Neta 2022). With this in mind, it does not take much modesty for Anna to admit that her book contains some errors. These considerations support our acceptance of (P2): she is justified in believing the preface claim.

The conclusion to be drawn from this discussion is that Anna's belief system is not unified. Anna serves as a counterexample to the Unified Picture, which says that all ordinary agents' belief systems are a unified whole. As mentioned, since the Consistency Principle is based on this false assumption, it is not normative for ordinary agents like Anna. So, it is wrong to accuse her of inconsistency. The outcome is that her belief system remains consistent.

<sup>&</sup>lt;sup>15</sup>See Foley (1987), p. 245 for this example.

<sup>&</sup>lt;sup>16</sup>I am grateful to Linda Radzik and Stephen Harrop for pressing me to justify my acceptance of (P1).

<sup>&</sup>lt;sup>17</sup>See Cherniak (1983), Weisberg (2020) for a defense of this claim. However, see Christensen (2004) for a suggestion on how to overcome this kind of cognitive limitation.

# 5. Anna's belief system is unified

In the last section, I argued that Anna's belief system lacks unity and this resulted in the conclusion that her belief system remains consistent. In this section, I assume for the sake of argument that Anna's belief system is unified. However, it will lead to the same conclusion. To see why, we just need to make a little adjustment to the above argument.

- (P1) If Anna's belief system is a unified whole, then she is not justified in believing the preface claim.
  - (P2') Anna's belief system is a unified whole.
  - (C') Therefore, she is not justified in believing the preface claim.

This reflects a popular philosophical motto: one person's modus ponens is another's modus tollens. If one disagrees with the conclusion from the last section (i.e., Anna's belief system is not unified), here is one's modus ponens. That is, if one thinks Anna's belief system is a unified whole, one should conclude that she has no reason to believe the preface claim.

This argument is similar to the one I presented in the last section. We have already covered (P1), and the reasons for accepting it remain applicable here. There is no need to repeat them. (P2 $^{\prime}$ ) is the assumption that we make in this section: Anna's belief system is unified. Taken together, we arrive at (C $^{\prime}$ ): Anna is not justified in believing the preface claim.

Note, however, that if her acceptance of the preface claim is not justified, then the preface paradox cannot even get off the ground. The paradox occurs because rationality demands that Anna believe the main text's assertions and the preface claim, while consistency suggests that doing so leads to inconsistency. This is often seen as a conflict between rationality and consistency. However, if Anna's belief system were unified, rationality would not require her to believe the preface claim at all, as previously argued for in (P1). Thus, no conflict would arise. My point is that if her belief system is unified, as proponents of the Unified Picture argue, it must also be consistent. The lack of justification for the preface claim should not create any inconsistency in her belief system.

When considering the preface paradox, we usually think of Anna writing a book that includes the preface claim, which leads to the problem. While the book has the preface claim and it does not align with the main text, the book is what it is. It is worth noting that the preface paradox only arises if Anna has *reason* to believe everything she asserts in the book, including the preface claim. However, if her belief system were unified, she would have no good reason to believe the preface claim. She should not have written it in the first place, or she must have made a mistake when she wrote it. Therefore, while there may be an inconsistency in the book itself, there is no inconsistency in her belief system, as we can all agree that her belief in the preface claim is unjustified.

## 6. Stating the result

The conclusion of this paper is proved by the following valid reasoning:

- 1.  $p \vee \neg p$
- 2.  $\neg p \rightarrow r$
- 3.  $p \rightarrow r$
- 3.  $r \lor r$
- 5. Therefore, *r*

Let us recap. In Section 2, it was observed that the Consistency Principle follows from the conjunction of two assumptions: Unified Picture: All ordinary agents' belief systems are a unified whole.

**Structural Requirement:** All components of a belief system are rationally required to be consistent.

Analyzing the principle in this way led us to question the accuracy of the Unified Picture. We then asked: is Anna's belief system unified? It must be either unified or not. This marks the first step:  $p \lor \neg p$ .

In Section 4, we showed that if Anna's belief system is not unified, then it would still be consistent. This corresponds to the second step:  $\neg p \rightarrow r$ . Here is why: if Anna's belief system is not unified, the Consistency Principle is not normative for her. Without the principle being relevant, there is no basis to label Anna's rational beliefs as inconsistent. Therefore, if Anna's belief system is not unified, it remains consistent.

In Section 5, we observed that if Anna's belief system is unified, then it would also be consistent. This corresponds to the third step:  $p \rightarrow r$ . The reasoning is as follows: if Anna's belief system is unified, then she would not be justified in believing (or making) the preface claim. Without rational acceptance of the preface claim, she would not break the Consistency Principle (i.e., its Structural Requirement in particular). Since she would not breach the principle, her belief system remains consistent.

The inference from the fourth step  $(r \lor r)$  to the fifth step (r) is trivially true. Putting everything together, the result is this: Anna's belief system remains consistent, regardless of whether it is unified or not. If this conclusion is correct, there is no reason to think that the Rationality Thesis and the Consistency Principle are in conflict. Thus, the preface paradox does not succeed in establishing the worrying conclusion that beliefs are individually rational but jointly inconsistent.

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