20

Drell–Yan process

It corresponds to the sub-process, where the quark and anti-quark come from the two scattering hadrons, and annihilate into vector bosons (photon, $W^\pm$, Z^0) with large invariant mass and then produce a lepton pair. A classical example is the annihilation into photon and with the production of e^+e^-:

$$\bar{q}q \to e^+e^- , \tag{20.1}$$

shown in Fig. 20.1. Drell–Yan process offers the possibility to test perturbative QCD as the large scale is given by the invariant mass of the lepton pair (of the order of $M_{W,Z}$ at CERN and Tevatron energies), while the parton densities enter quadratically in this process where the final state is totally inclusive.

20.1 Kinematics

The kinematics of the process is characterized by the parton distribution $q_f^{h_i}(x)$ for a quark of flavour f issued from the hadron h_i. The total momentum squared of the subprocess is:

$$Q^2 = (x_1 p_1 + x_2 p_2)^2 , \tag{20.2}$$

and coincides with the invariant mass squared of the photon. The total energy squared of the hadron is:

$$s = (p_1 + p_2)^2 . \tag{20.3}$$

For large s, one usually neglects the hadron mass, such that one can approximately write:

$$Q^2 \simeq x_1 x_2 s . \tag{20.4}$$

Another useful variable is:

$$x_F \equiv x_1 - x_2 , \tag{20.5}$$

and the *rapidity* y defined as:

$$\tanh y = \frac{x_1 - x_2}{x_1 + x_2} \quad \text{or} \quad y = \frac{1}{2} \ln \frac{x_1}{x_2} . \tag{20.6}$$

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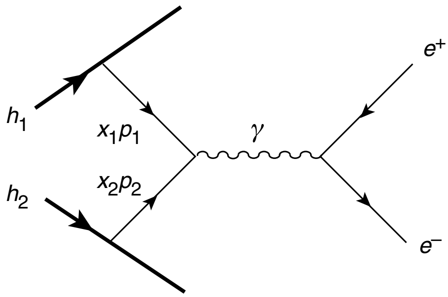

Fig. 20.1. Drell–Yan process.

Alternatively, in the hadron-hadron centre of mass where the photon momentum is:

$$q = (E; q_\parallel, q_\perp) , \tag{20.7}$$

one has:

$$x_F = 2q_\parallel/\sqrt{s} , \qquad y = \frac{1}{2}\ln\frac{E+q_\parallel}{E-q_\parallel} . \tag{20.8}$$

20.2 Parton model

20.2.1 Cross-section

In order to evaluate the production cross-section, one calculates the reduced cross-section corresponding to the subprocess in Eq. (20.1), and write the total cross-section as a convolution. Neglecting quark and electron masses, the point-like cross-section reads, to lowest order:

$$\hat\sigma_{l.o}(\bar q + q \to e^+e^-) = \frac{4\pi\alpha^2 Q_f^2}{3N_c Q^2} , \tag{20.9}$$

where Q_f is the quark charge in units of e. The full lowest order differential cross-section reads:

$$\frac{d\sigma_{l.o}}{dQ^2} = \frac{4\pi\alpha^2}{3N_cQ^2}\sum_f Q_f^2 \int_0^1 \frac{dx_1}{x_1}\int_0^1\frac{dx_2}{x_2}\delta(1-z)\left[q_f^{h_1}(x_1)\bar q_f^{h_2}(x_2) + \bar q_f^{h_1}(x_1) q_f^{h_2}(x_2)\right] , \tag{20.10}$$

where:

$$\tau \equiv Q^2/s \quad \text{and} \quad z \equiv \frac{\tau}{x_1x_2} . \tag{20.11}$$

τ quantifies the fraction of energy squared that goes into the lepton pair. If τ is small, then, one of the x_i is small and then favours the sea quark contribution. If the x_i is maximal i.e. around $1/3 \sim 1/4$, then the valence contribution will dominate. The Drell–Yan processes are important as they can provide a non-trivial test of the validity of the parton approach and of its extension in QCD through the factorization theorem. One expects that the parton

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densities measured in lepto-production for a given hadron target should be relevant to make predictions on the Drell–Yan and some other DIS processes.

20.2.2 Approximate rules

There are typical rules for Drell–Yan processes.

Intensity rules

From the above-mentioned properties, one expects that, for large x_i, the cross-section involving two valence quarks for producing the e^+e^- pair, is much larger than the one involving one valence and one sea quarks. For an isoscalar target one, e.g., expects:

$$\frac{\sigma(\pi^+ N(I=0))}{\sigma(\pi^- N(I=0))} \to \frac{1}{4} \,. \tag{20.12}$$

Scaling

In the region where the naïve parton model is valid, one expects that the dimensioneless quantities:

$$Q^4 \frac{d\sigma}{dQ^4} \,, \qquad Q^4 \frac{d\sigma}{dQ^2 dx_F} \,, \qquad Q^4 \frac{d\sigma}{dQ^2 dy} \,, \tag{20.13}$$

should scale as functions of the scaling variables $\tau,\ x_F$ and y independently of Q^2.

Angular distribution of leptons

For large Q^2, where the longitudinal structure function (W_L) is much smaller than the transverse (W_T) one, the lepton pair angular distribution originated from an off-shell photon is predominantly of the form:

$$\frac{d\sigma}{dQ^2 d\cos\theta} \sim W_T(Q^2, \tau)(1+\cos^2\theta) \,. \tag{20.14}$$

Atomic number

The cross-section being proportional to the number of quarks or antiquarks in the target nucleus, each contribution adding up incoherently, one expects a linear dependence with the atomic number A in the Drell–Yan region.

20.3 Higher order corrections to the cross-section

The different processes relevant to the NLO corrections are:

$$\begin{aligned} &q+\bar{q} \to \gamma^* \\ &q+\bar{q} \to \gamma^* + g \\ &g + q(\text{or } \bar{q}) \to \gamma^* + q(\text{or } \bar{q}) \,, \end{aligned} \tag{20.15}$$

where γ^* produces the lepton pairs e^+e^-. They are shown in Fig. 20.2.

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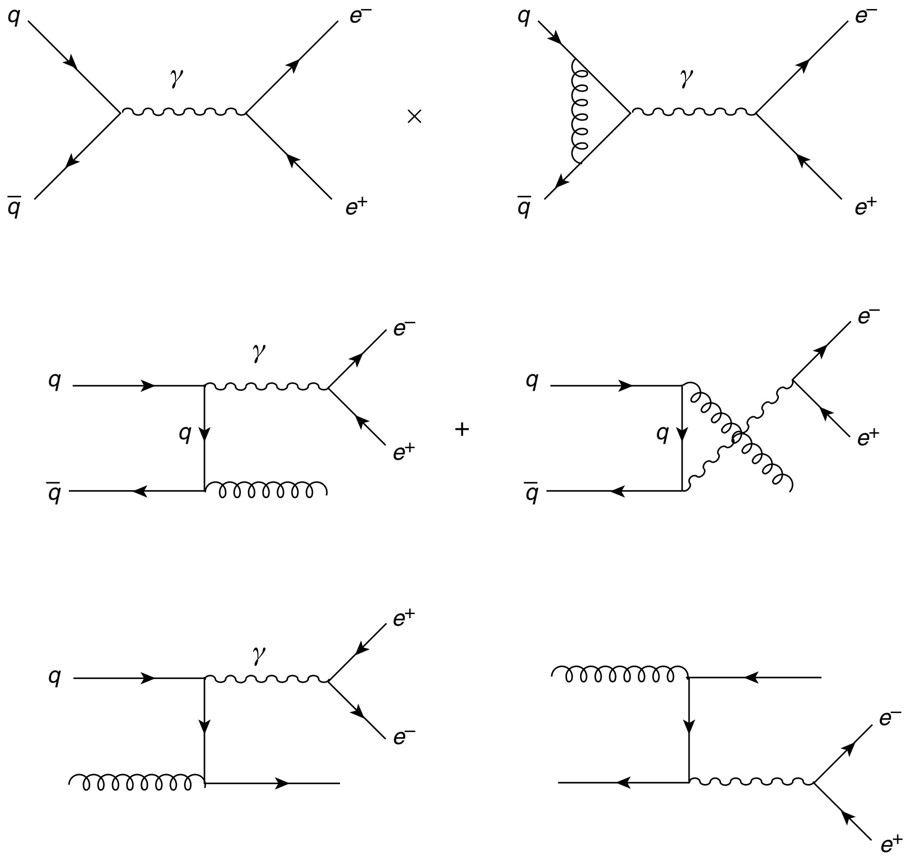

Fig. 20.2. NLO corrections to the Drell–Yan process.

Technically, the evaluation of higher order corrections is not easy because of the interplay between the IR and mass singularities. The NLO corrections have been obtained in [270], and the NNLO corrections in [271]. The interactions with the spectator quarks induce a $1/Q^2$ power corrections analogue of the higher twist term in DIS. The expression of the cross-section including the NLO corrections reads:

$$\frac{d\sigma_{l.o}}{dQ^2} = \frac{4\pi\alpha^2}{3N_c Q^2} \sum_f Q_f^2 \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \left\{ \left[\delta(1-z) + \left(\frac{\alpha_s}{\pi}\right) \theta(1-z) \Phi_q(z) \right] \right.$$
$$\times \left[q_f^{h_1}(x_1) \bar{q}_f^{h_2}(x_2) + \bar{q}_f^{h_1}(x_1) q_f^{h_2}(x_2) \right] ,$$
$$\left. + \left(\frac{\alpha_s}{\pi}\right) \theta(1-z) \Phi_g(z) \left[q_f^{h_1}(x_1) + \bar{q}_f^{h_2}(x_1) \right] g^{h_2}(x_2, Q^2) + (1 \leftrightarrow 2) \right\} , \qquad (20.16)$$

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where:

$$\Phi_q(z) = \frac{C_F}{2}\left[\frac{3}{(1-z)_+} - 6 - 4z + 2(1+z^2)\frac{\ln(1-z)}{(1-z)_+} + \left(1+\frac{4\pi^2}{3}\right)\delta(1-z)\right],$$

$$\Phi_g(z) = \frac{1}{2}\left[[z^2+(1-z)^2]\ln(1-z) + \frac{9z^2}{2} - 5z + \frac{3}{2}\right]. \tag{20.17}$$

In the case of $\bar{p}p$ collisions, the valence quarks and antiquarks contribution dominates in the Drell–Yan region. In the case of pp collisions, the anti-quark comes from the sea such that the contribution of the anti-quark and of the gluon are comparable.

20.4 The K factor

Noting that the correction term proportional to $\delta(1-z)$ comes from vertex corrections and from a radiation of zero momentum gluons, which cancels the IR singularity in the vertex, one can separate this term from the others and rewrite:

$$\delta(1-z) + \left(\frac{\alpha_s}{\pi}\right)\Phi_q(z) \equiv K_{\text{vertex}}\delta(1-z) + \left(\frac{\alpha_s}{\pi}\right)\Phi_q(z)_{\text{reg}} \tag{20.18}$$

where $\Phi_q(z)_{\text{reg}}$ is the regular part of $\Phi_q(z)$ and:

$$K_{\text{vertex}} = 1 + \frac{C_F}{2}\left(1+\frac{4\pi^2}{3}\right)\left(\frac{\alpha_s}{\pi}\right). \tag{20.19}$$

One can notice that the radiative corrections in the regular part of the cross-section are small. The most important correction comes from the π^2 part of K_{vertex}, where it has been noticed [272] that part of this large correction can be resummed and exponentiates:

$$1 + C_F\frac{\pi^2}{2}\left(\frac{\alpha_s}{\pi}\right) \to K(Q^2) \equiv \exp\left(\frac{C_F}{2}\pi\alpha_s\right), \tag{20.20}$$

while the remaining correction:

$$1 + \frac{C_F}{2}\left(1+\frac{\pi^2}{3}\right)\left(\frac{\alpha_s}{\pi}\right), \tag{20.21}$$

is comfortably small. However, one should be aware of the fact that the resummation procedure is not unique. Different phenomenology of the Drell–Yan processes have been performed at Tevatron, which can be consulted from different contributions at various conferences, like the QCD-Montpellier series.